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TESTING OF

ELECTRO-MAGNETIC MACHINERY

AND OTHER APPARATUS

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BY

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VOLUME II.

ALTERNATING CURRENTS

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To the Memory of

PROFESSOR JOHN BUTLER JOHNSON, C.E.

PAST DEAN OF THE COLLEGE OF ENGINEERING
UNIVERSITY OF WISCONSIN.

"The efficient direction of any industry to-day demands a very large amount of technical knowledge which cannot be learned at the bench or in the shops."

J. B. Johnson.

PREFACE.

At the time of the publication of Volume I, dealing with direct current electro-magnetic machinery and apparatus, a second volume relating to the testing of alternating current machinery and apparatus was promised. At that time a set of ninety-three alternating current experiments, written simultaneously with those on direct current contained in Volume I, was in existence and had gone through several editions in the form of mimeograph notes for use in the laboratories of the University of Wisconsin. Owing to the rapid development of alternating current theory and apparatus then going on, the authors did not feel warranted in publishing the present volume at that time and therefore planned to put it through three more revisions in as many years before publication. In the six years which have elapsed time was not found in which to complete the work, but the notes have been used and revised at the Universities of Wisconsin and Illinois by the respective laboratory forces. In 1909 and 1910, Professor J. M. Bryant, of the University of Illinois, assumed the major portion of the work involved in completing the manuscript. In doing this, he revised a large number of the experiments and added much new material.

Alternating current development is now on a par with direct current development. Professor Bryant has kept the work in close touch with teaching experience and, with the experience of the authors of Volume I in the practice of electrical engineering in the great engineering centres of the United States, it is felt that Volume II is now ready for publication.

The same plan has been followed here as in Volume I. Each experiment is self-contained and has been considered in the same general manner. In each case a number of **References** is furnished for the purpose of more thoroughly familiarizing the stu-

dent with the subject under consideration before performing the experiment and to assist him in the preparation of his written report. Considerable space is devoted to **Theory and Method;** the observations required are specified under **Data;** in certain cases **Curves** are to be drawn; where necessary, **Cautions** are given to avoid accidents or a possible repetition of the experiment; and, finally, **Questions** bearing directly upon the subject, are asked.

Some criticism has been made that the employment of the paragraph **Data** tends to coddle the student. The experience of the authors is that engineering students are men, well beyond the coddling age in matters of this kind, and that the saving in time and unnecessary repetition of experiments which this paragraph effects is fully justified.

In the arrangement of the experiments a rough classification has been made and certain groups of experiments have been provided with short introductions to give the student a general idea of the method used by the authors in dealing with that particular group.

No definite sequence of experiments has been planned, as the student is largely dependent on the course he has had in the lecture room, on the equipment of the laboratory and on the ability of the instructor to use that equipment to best advantage, for the order in which he performs the experiments. It is doubtful whether any university laboratory will ever be so fully equipped with duplicate apparatus that undergraduates can pursue a definite sequence of experiments. This would be a bad thing for the student; he gets ample sequence in the class room and there is no tabulated sequence of life's problems after commencement day.

Acknowledgment is due to Professor Murray C. Beebe, of the University of Wisconsin, who collaborated with one of the authors in the preparation of the first set of mimeographed notes, and to Professor John W. Schuster, also of the University of Wisconsin, who has used the mimeographed notes in his classes for the past

ten years and who has given the authors many valuable suggestions. Professor Dugald C. Jackson, of the Massachusetts Institute of Technology, Professor Ernst J. Berg, of the University of Illinois, and Professor Morgan Brooks, also of the University of Illinois, have shown a friendly interest in the progess of this work and have materially assisted the authors in various ways.

The authors are under obligations to Mr. Alexander M. Gray, of the electrical engineering staff of the Allis-Chalmers Company, for his great assistance in criticising the manuscript of many of the experiments and in offering suggestions which have materially increased the value of the book from the standpoint of bringing the student closer to the problems which confront the engineer when engaged in the commercial testing and operation of alternating current machinery.

It had been intended to write an appendix on resuscitation, but the subject has been so thoroughly handled by Mr. Clem. A. Copeland in his pamphlet that permission was obtained to use it. Not only has he treated of resuscitation, but the whole subject of life hazard to electrical workers is analyzed and rules for its minimization are offered. Thanks are extended to Mr. Copeland and to his publisher, Mr. E. B. Strong, for permission to use the pamphlet on resuscitation, the major portion of which is reproduced as **Appendix A** of the present volume.

While this work has been carefully edited, the authors do not presume that all errors have been eliminated and they will be grateful if any such are called to their attention.

December, 1910.

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NOMENCLATURE.

A,	Area.
B,	Susceptance.
$\mathfrak{B}$,	Magnetic Induction.
C, c,	Capacity.
D, d,	Unknown Quantity.
E,	Effective Value of Electromotive Force.
e,	Instantaneous Value of Electromotive Force.
E_1, E_2, E', etc.,	Definite Vector Values of Electromotive Force.
F, f,	Force.
f,	Frequency.
$\mathfrak{F}$,	Magneto-motive Force.
G, g,	Conductance.
$\mathfrak{H}$,	Magnetizing Force.
I,	Effective Value of Current.
i,	Instantaneous Value of Current.
I_1, I_2, I', etc.,	Definite Vector Values of Current.
$\mathfrak{I}$,	Intensity of Magnetization.
K,	Moment of Inertia.
K, k,	Constant.
K,	Ratio of Transformation.
L,	Coefficient of Inductance.
l,	Length.
M,	Mass.
m,	Strength of Pole.
$\mathfrak{M}$,	Magnetic Moment.
N, n,	Number of Turns.
NI, ni,	Ampere Turns.
O,	Origin.
P,	Power (Electrical or Mechanical).
p,	Pressure, Pull, Power Factor.

p_1, Number of Pairs of Poles.
p_2, Number of Pairs of Armature Paths.
Q, q, Quantity.
q, Reactance Factor.
R, r, Resistance.
$\mathfrak{R}$, Magnetic Reluctance.
S, Number of Armature Conductors.
s, Surface.
T, Torque, Time Constant, Duration of a Cycle.
t, Time, Temperature.
U, u, Unknown Quantity.
V, Volume, Revolutions per Minute.
v, Velocity, Leakage Coefficient.
W, Weight.
W, w, Work, Electrical Energy.
X, x, Reactance.
Y, y, Admittance.
Z, z, Impedance.
α, Acceleration, Angle.
β, Angle.
γ, Conductivity.
δ, Deflection.
ϵ, Base of Naperian Logarithms.
η, Efficiency, Hysteresis Coefficient.
θ, Angle, Deflection, Temperature Coefficient.
κ, Magnetic Susceptibility.
μ, Magnetic Permeability.
π, Ratio of the Circumference of a Circle to its Diameter.
ρ, Magnetic Reluctivity.
σ, Resistivity.
τ, Time Constant.
ϕ, φ, Magnetic Flux.
ω, Angular Velocity $= 2\pi f$.

LIST OF REFERENCES.

1. *Am. Elect'n.* *American Electrician.* m. New York.
2. *Am. Jour. Sci.* *American Journal of Science.* m. New Haven, Conn.
3. ARNOLD. Die Wechselstromtechnik. By E. Arnold. Vols. 1 to 5. Berlin, 1902 to 1909.
4. BEDELL. Principles of the Transformer. By Frederick Bedell. 1907.
5. BEDELL AND CREHORE. Alternating Currents. By Frederick Bedell and A. C. Crehore. Third Edition. 1895.
6. BEHREND. The Induction Motor, its Theory and Design. By B. A. Behrend. 1901.
7. BERG. Electrical Energy. By Ernst Julius Berg. 1908.
8. BOY DE LA TOUR. The New Induction Motor. By Henri Boy de la Tour. 1903.
9. *Bull. Bu. Standards.* *Bulletin of the United States Bureau of Standards.* m. Washington, D. C.
10. *Bull. Univ. Wis.* *Bulletin of the University of Wisconsin.* m. Madison, Wis.
11. *Can. Elec. News.* *Canadian Electrical News.* m. Toronto.
12. CARHART AND PATTERSON. Electrical Measurements. By Henry S. Carhart and George W. Patterson, Jr. 1895.
13. *Ecl. Elec.* *Eclairage Electrique.* w. Paris.
14. *Elec. Eng. Lond.* *Electrical Engineer.* w. London.
15. *Elec. Jour.* *Electric Journal.* m. Pittsburgh, Pa.
16. *Elec. Rev.* *Electrical Review.* w. New York.
17. *Elec. Rev. Lond.* *Electrical Review.* m. London.
18. *Elect'n Lond.* *Electrician.* w. London.
19. *Elec. Times.* *Electrical Times.* w. London.
20. *Elec. Wld.* *Electrical World.* w. New York.
21. *Elec. Wld. and Eng.* *Electrical World and Engineer.* w. New York.

22. *Elektrotech. u. Maschinenbau. Electrotechnik und Maschinenbau.* w. Vienna.

23. *Elektrotech. Zeitschr. Elektrotechnische Zeitschrift.* m. Berlin.

24. *Eng. Rec. Engineering Record.* w. New York.

25. ESTY. Alternating Current Machinery. By William Esty. 1909.

26. FLEMING. Handbook for the Electrical Laboratory and Testing Room. By J. A. Fleming. Vol. 1, 1901. Vol. 2, 1903.

27. FLEMING'S "Transformers." Alternate Current Transformers, in Theory and Practice. By J. A. Fleming. Third Edition. 1901.

28. FRANKLIN AND ESTY. Elements of Electrical Engineering. By William S. Franklin and William Esty. 1907.

29. GERHARDI. Electricity Meters. By C. H. W. Gerhardi. 1907.

30. GRAY. Absolute Measurements in Electricity and Magnetism. By Andrew Gray.

31. *Handbuch der Elektrotech. Handbuch der Elektrotechnik.*

32. *Harvard Engng. Jour. Harvard Engineering Journal.* q. Cambridge, Mass.

33. HAY. Alternating Currents, their Theory, Generation and Transformation. By Alfred Hay. 1905.

34. JACKSON. Alternating Currents and Alternating Current Machinery. By D. C. Jackson and J. P. Jackson. Revised Edition. Part I. 1910.

35. *Jour. Electricity, Power and Gas. Journal of Electricity, Power and Gas.* m. San Francisco, Cal.

36. *Jour. Fr. Inst. Journal Franklin Institute.* m. Philadelphia, Pa.

37. *Jour. Worcester Poly. Inst. Journal of Worcester Polytechnic Institute.* b.m. Worcester, Mass.

38. KAPP. Dynamos, Motors and Transformers. By Gisbert Kapp. 1902.

39. KARAPETOFF. Experimental Electrical Engineering. By V. Karapetoff. First Edition. 1908.

40. LAMB. Alternating Currents. By C. G. Lamb, 1906.
41. MCALLISTER. Alternating Current Motors. By Addams Stratton McAllister. Third Edition. 1909.
42. *Mech. Eng. Mechanical Engineer.* w. Manchester, England.
43. Manufacturers' Bulletins. Commercial Bulletins of Various Manufacturers of Electrical Machinery and Apparatus.
44. *Min. and Sci. Pr. Mining and Scientific Press.* w. San Francisco, Cal.
45. PARSHALL AND HOBART. Armature Windings of Electric Machines. By H. F. Parshall and H. M. Hobart. Second Edition.
46. *Prac. Eng. Practical Engineer.* w. London.
47. *Pro. Inst. Elec. Eng. Proceedings of the Institute of Electrical Engineers.* q. London.
48. RUSSELL. A Treatise on the Theory of Alternating Currents. By A. Russell. London. 1904 and 1905.
49. RYAN. Text Book of Electrical Machinery. By H. J. Ryan, H. H. Norris and G. L. Hoxie. 1904.
50. *School of Mines Qr. School of Mines Quarterly.* qr. New York, N. Y.
51. SEVER AND TOWNSEND. Laboratory and Factory Tests in Electrical Engineering. By G. Sever and Fitzhugh Townsend. Second Edition. 1907.
52. Standard Handbook. The Standard Handbook for Electrical Engineers. Second Edition. 1908.
53. *Sib. Jour. Engng. Sibley Journal of Engineering.* m. Ithaca, N. Y.
54. STEINMETZ' "A.C. Phenomena." Alternating Current Phenomena. By C. P. Steinmetz. Fourth Edition. 1908.
55. STEINMETZ' "Elements." Theoretical Elements of Electrical Engineering. By C. P. Steinmetz. Third Edition. 1909.
56. STEINMETZ' "Radiation." Radiation, Light and Illumination. By C. P. Steinmetz. 1909.
57. STEINMETZ' "Transient Phenomena." Theory and Calcula-

tion of Transient Electric Phenomena and Oscillations. By C. P. Steinmetz. 1909.

58. *Stevens Ind. Stevens Institute Indicator.* qr. Hoboken, N. J.
59. STEWART AND GEE. Lessons in Elementary Practical Physics. Vol. 2, Electricity and Magnetism. By Balfour Stewart and W. W. Haldane Gee.
60. *St. Ry. Jour. Street Railway Journal.* w. New York.
61. SWENSON AND FRANKENFIELD. Testing of Electro Magnetic Machinery and other Apparatus. By B. V. Swenson and Budd Frankenfield. Vol. 1.
62. *Tech. Technograph.* yr. Urbana, Illinois.
63. *Tech. Qr. Technology Quarterly.* qr. Boston, Mass.
64. THOMÄLEN. A Text-Book of Electrical Engineering. By A. Thomälen. Translated by George W. O. Howe. 1907.
65. THOMPSON'S "Dynamos." Dynamo Electric Machinery. By S. P. Thompson. Seventh Edition.
66. THOMPSON'S "Polyphase." Polyphase Electric Currents and Alternate Current Motors. By S. P. Thompson. 1895.
67. *Trans. Am. Inst. Elec. Eng. Transactions of the American Institute of Electrical Engineers.* yr. New York.
68. *Trans. Int. Elec. Cong. Transactions of the International Electrical Congress.* St. Louis, 1904.
69. *Zeitschr. f. Elektrotech. Zeitschrift für Elektrotechnik.* w. Vienna.

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2. IMPEDANCE OF A CIRCUIT CONTAINING A NON-INDUCTIVE RESISTANCE AND CAPACITY IN SERIES.
3. MEASUREMENT OF CAPACITIES IN PARALLEL AND IN SERIES BY THE IMPEDANCE METHOD.
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6. VALUE OF CURRENT AND PHASE ANGLE IN A CIRCUIT OF VARYING CAPACITY; THE RESISTANCE, INDUCTANCE, PRESSURE AND FREQUENCY REMAINING CONSTANT.
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12. REGULATION OF A TRANSMISSION LINE CONTAINING NEGLIGIBLE INDUCTANCE AND CAPACITY.
13. REGULATION OF A TRANSMISSION LINE CONTAINING RESISTANCE AND INDUCTANCE BUT NEGLIGIBLE CAPACITY.

PRELIMINARY.

If the student has had a vacation since his work in the direct current laboratory, it will be a good plan to review the matter under **Preliminary** in Volume I. There is little to add to this heading in the present volume.

In the design of this manual the circuit and the transmission line are studied first, in order to fix the fundamentals of methods which are used in the study of machines and apparatus. Wherever practicable, the machine or other apparatus is then studied as a special case of an elementary circuit.

The graphical method has been introduced in most of the experiments, as experience the world over has fixed this as the best method of solving engineering problems, either in the class room or in the practice of the profession. It has its limitations, but the student should now be far enough along in his course to recognize them. This method will be found inaccurate in the solution of quantitative problems in cases where there is great variation in magnitude of the components of a diagram but even here the problem is easier to understand when represented graphically, though the actual solution may be effected analytically by rectangular components or by symbolic notation. As in other alternating current text books, the simple sine wave is assumed and all problems are represented in plane geometry. The representation of a phenomenon in higher harmonics requires three dimensions and it is fortunate that the effect of higher harmonics in alternating current applications can generally be neglected.

Circuits are classified as series, parallel and complex. Current is chosen as the reference axis in series circuits, while pressure is so considered in parallel circuits. In complex circuits, the parallel portions are usually treated first and the total current is derived in reference to the pressure across the branches, this current being that in the series portion of the circuit.

The student is advised to read the short introductions preceding the experiments to which they refer; they have special reference to the treatment of the subjects in this manual. No attempt has been made to outline a theory of alternating currents, but an ample bibliography has been included with nearly every experiment, which should enable the student to read up on theory and application either before performing the experiment or before writing the report. The work he has had in the class room is depended upon for his basic theory.

It is designed to place the student more on his own resources in the treatment of Volume II than was the intention in Volume I, and he should do his utmost to develop his originality. Many of the experiments can be performed in another way; some may be varied, while others suggest experiments not in the list. The student is therefore in splendid opportunity to develop his creative side.

ELECTRO-MAGNETIC MACHINERY

No. 1. DETERMINATION OF REACTANCE BY THE IMPEDANCE METHOD; STUDY OF A CIRCUIT CONTAINING RESISTANCE AND INDUCTANCE IN SERIES.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 7, Art. 159; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2^4, pp. 24 to 79.

Object. To determine the reactance of a simple series circuit containing resistance and inductance, and to show the phase relation existing between the current and the various pressures.

Theory and Method. In a series circuit containing resistance and inductance the value of the current is

$$I = \frac{E}{\sqrt{(R+r)^2 + (2\pi f L)^2}} = \frac{E}{\sqrt{(R+r)^2 + X^2}} = \frac{E}{Z}, \qquad (1a)$$

where R = the resistance of the non-inductive part of circuit,
r = the resistance of the inductive part of circuit,
L = the inductance of the inductive part of circuit, and
Z = the impedance of the circuit (see Figures 1D and 1F).

When an alternating electromotive force is impressed upon the circuit, its phase angle is

$$\beta = \tan^{-1}\frac{2\pi f L}{R+r}. \qquad (1b)$$

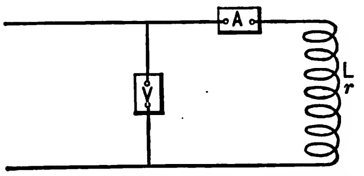

FIG. 1*A*. Connections for measuring the impedance of a coil.

Equation (1*a*) shows a convenient method of determining the value of X for such a circuit by a method analogous to the fall of potential method of measuring resistances. The value of inductance may also be determined by this method.

Equation (1*a*) solved for X will give

$$X = \sqrt{\left(\frac{E}{I}\right)^2 - (R+r)^2}.$$

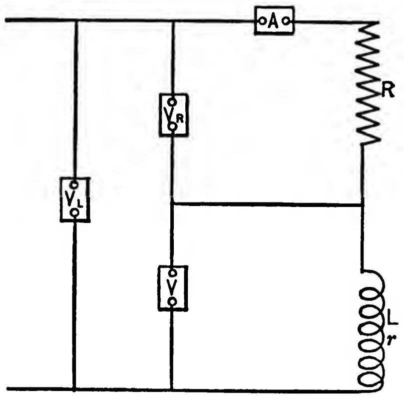

FIG. 1*B*. Connections for measuring the impedance of a circuit containing resistance and inductance in series.

If the resistance of the coil be measured and readings taken as indicated in Figures 1*A* or 1*B*, the value of the reactance may be computed. Voltage and impedance diagrams may be drawn as in Figures 1*C* and 1*D*, showing the phase relations between the current and the voltage drop across each part of the circuit. If the frequency is also measured, the value of the inductance may be computed.

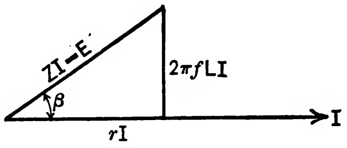

FIG. 1*C*. Electromotive force diagram for a single inductance.

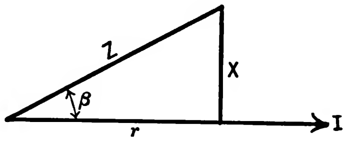

FIG. 1*D*. Impedance diagram for a single inductance.

Data. Using an inductance without an iron core, connect as in Figure 1*A*, taking several observations at different frequencies and pressures, if possible. Also obtain data for computing the resistance r of the coil. Place a non-inductive resistance R in

series with the coil, and take a set of readings as in Figure 1*B*. Vary the value of the resistance and take a set of readings for each value. Determine the frequency of the circuit.

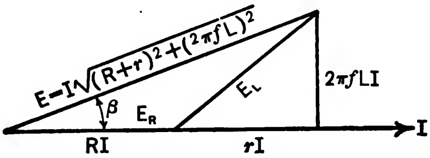

FIG. 1*E*. Electromotive force diagram, resistance and inductance in series.

Graphical Solution. Compute the values of *R, r, X, Z* and lag angle for the complete circuit, and draw voltage and impedance diagrams as indicated in Figures 1*C*, 1*D*, 1*E* and 1*F*.

The voltage diagrams may be constructed from the actual readings, if the drop across a non-inductive resistance produced by the

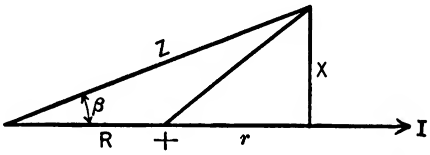

FIG. 1*F*. Impedance diagram, resistance and inductance in series.

same current is known, as these values form the three sides of a triangle with the resistance drop in phase with the current axis (see Figure 1*E*).

Curves. Plot curves showing the variation of *Z* and *I* with variation of *R* over as wide a range as possible, using *R* as abscissas (see Figure 1*F*).

No. 2. IMPEDANCE OF A CIRCUIT CONTAINING NON-INDUCTIVE RESISTANCE AND CAPACITY IN SERIES.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff,

Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 7, Art. 159; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2[4], pp. 24 to 79.

Object. To measure the impedance of a condenser, to calculate its capacity, and to measure the impedance of a simple series circuit containing a capacity and a non-inductive resistance.

Theory and Method. In a series circuit containing resistance and capacity, the value of the current is

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}} = \frac{E}{\sqrt{R^2 + X^2}} = \frac{E}{Z}, \qquad (2a)$$

and its phase angle of advance is determined from the equation

$$\beta = \tan^{-1}\left(-\frac{1}{2\pi f C R}\right),$$

as seen in Figures $2B$ and $2C$.

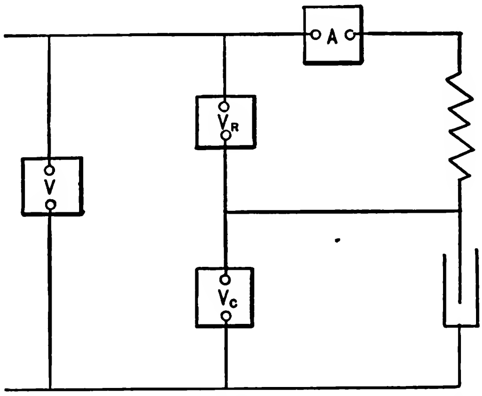

FIG. 2*A*. Connections for measuring the impedance of a circuit containing resistance and capacity in series.

Data. Connect the capacity and non-inductive resistance in series, as indicated in Figure 2*A*. Take several observations at different pressures and frequencies, if possible.

Graphical Solution. The graphical solution should be worked out as shown in Figures *2B* and *2C*. In this case it must be remembered that the current has an angle of advance instead of an angle of lag. Figure *2B* may also be laid off from the relation of the three pressure readings, E, E_C and E_R.

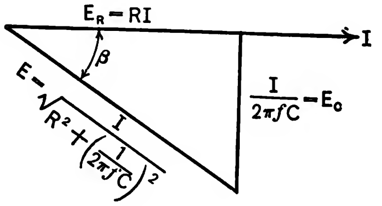

FIG. *2B*. Electromotive force diagram, resistance and capacity in series.

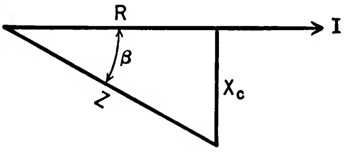

FIG. *2C*. Impedance diagram, resistance and capacity in series.

Curves. Plot curves showing the variation of Z and I with variation of R, over as wide a range as possible, using R as abscissas.

Explain. What conditions arise when the capacity becomes infinite? When the frequency is zero? What is the effect of variation of frequency upon the impedance of the circuit and its angle of lead?

No. 3. MEASUREMENT OF CAPACITIES IN PARALLEL AND IN SERIES BY THE IMPEDANCE METHOD.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8, Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2[4], pp. 24 to 79.

Object. To determine the total capacity of a group of condensers connected either in series or in parallel.

Theory and Method. The theoretical law for the capacity of

a group of condensers, when connected in parallel, is that their combined capacity is equal to the sum of the individual capacities. When connected in series the reciprocal of the combined capacity is equal to the sum of the reciprocals of the individual capacities. These laws are similar to those for the combined resistance of several individual resistances when placed in series and in parallel, respectively.

A physical conception of these laws may be had by imagining two condensers, exactly alike, connected in parallel; the effect will be that of increasing the area of the plates of one condenser to twice the original area and the capacity will be doubled. When the two condensers are connected in series, the effect is the same as if the plates of one of the condensers were separated to twice the original distance and the capacity will be halved.

In this experiment several capacities are to be measured separately by the impedance method, and then their combined capacity is to be measured by the same method when connected in series, and also when connected in parallel. From the equation

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}, \qquad (3a)$$

the value of C may be computed. In the ordinary condenser the loss is so small that R may be neglected and we have

$$I = 2\pi f C E,$$

whence,

$$C = \frac{I}{2\pi f E}. \qquad (3b)$$

Data. Take readings of E, I and frequency with various series and parallel combinations of condensers.

Caution. The frequency should be determined with considerable accuracy during the readings, otherwise the results will not check.

Compute. The value of C for each set of readings and check

against the value as computed from the separate capacities. If possible, check this value by some method of comparison with a standard condenser.

Question. What would be the effect of a variation of the pressure wave form between the different sets of readings?

No. 4. IMPEDANCE OF A CIRCUIT CONTAINING RESISTANCE, INDUCTANCE AND CAPACITY IN SERIES.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8, Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2^4, pp. 24 to 79.

Object. To investigate a circuit containing resistance, inductance and capacity in series.

Theory and Method. When an alternating pressure is impressed upon a circuit containing resistance, inductance and capacity in series, the current is

$$I = \frac{E}{\sqrt{(R+r)^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}} = \frac{E}{\sqrt{(R+r)^2 + X^2}} = \frac{E}{Z}, \quad (4a)$$

and the phase angle of the circuit (Figure $4B$) is expressed by the value

$$\beta = \tan^{-1} \frac{2\pi f L - \frac{1}{2\pi f C}}{R+r}, \quad (4b)$$

where R is the resistance of the non-inductive part of the circuit and r is that of the inductive part of the circuit. A positive value indicates an angle of lag, a negative value an angle of advance.

The various pressures may be obtained by observation. The total pressure should check with the value obtained by solving for E in the equation for current. The pressure across the non-inductive resistance will equal RI, and that across the inductive resistance will equal

$$E_L = I\sqrt{r^2 + (2\pi f L)^2},$$

also,

$$E_C = \frac{I}{2\pi f C}.$$

As these pressures have different phase angles in relation to the current, the total pressure may in many cases be less than either the pressure across the capacity or the inductive portion of the circuit, but it is always the vector sum of the pressures across the resistance, capacity and inductance. It is always equal to or greater than the pressure across the resistance. When the capacity and inductance effects are neutralized, that is, when

$$2\pi f L = \frac{1}{2\pi f C},$$

the circuit is said to be in resonance, and the current and impressed pressure are in phase with each other.

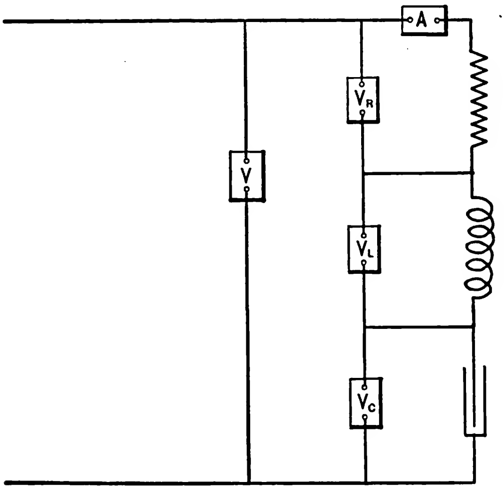

Fig. 4*A*. Connections for measuring the impedance of a circuit containing resistance, inductance and capacity in series.

Data. Readings of the total pressure, as well as the pressures across the various parts of the circuit, should be taken, as indicated in Figure 4*A*. These observations may be taken at various frequencies or with several different pressures. Also obtain data for computing the resistance r of the inductive circuit.

Graphical Solution. The pressure and impedance diagrams should be worked out in a manner similar to Figures 4*B* and 4*C*. The various pressures will be in phase with, in advance of, or will lag behind the current, according as the effect is that due to resistance, self-inductance or capacity. Figure 4*B* may be constructed from the voltmeter readings.

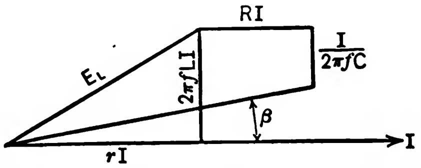

FIG. 4*B*. Electromotive force diagram, resistance, inductance and capacity in series.

Suggestion. It is well to have a capacity sufficiently large so that the condition

$$C = \frac{I}{(2\pi f)^2 L},$$

which is the condition of resonance, will be obtained.

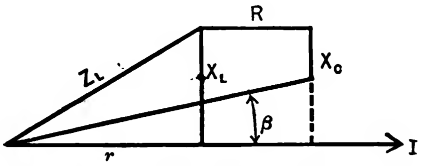

FIG. 4*C*. Impedance diagram, resistance, inductance and capacity in series.

Explain. When a circuit is in resonance at a given frequency, what is the effect of a slight increase or decrease in the frequency?

No. 5. VALUE OF CURRENT AND PHASE ANGLE IN A SERIES CIRCUIT OF VARYING INDUCTANCE; THE RESISTANCE, CAPACITY, PRESSURE AND FREQUENCY REMAINING CONSTANT.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8, Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2^4, pp. 24 to 79.

Object. To vary the value of inductance, the frequency, resistance, capacity and pressure remaining constant, and to note the change in the values of the current and the phase angle.

Theory and Method. When an alternating current flows in a series circuit containing resistance, inductance and capacity in series, the value of the current is

$$I = \frac{E}{\sqrt{(R+r)^2 + \left(2\pi fL - \dfrac{1}{2\pi fC}\right)^2}}. \tag{5a}$$

It may be seen from this that, when the resistance, capacity and frequency are kept constant, the current becomes a maximum when the value of L is such that

$$L = \frac{1}{(2\pi f)^2 C}. \tag{5b}$$

When this condition is satisfied, the current is

$$I = \frac{E}{R+r}, \tag{5c}$$

as in a non-inductive circuit.

The value of the phase angle is

$$\beta = \tan^{-1} \frac{2\pi fL - \dfrac{1}{2\pi fC}}{R+r}, \tag{5d}$$

and this becomes zero at the point of resonance. From this it may be seen that the current becomes a maximum and in phase with the electromotive force, at the point of resonance.

It may also be seen that, when L is greater than the fraction in equation 5*b*, β becomes positive and is an angle of lag. At resonance, it will be found that the drop across the inductance (and also across the capacity) may be much greater than the total voltage impressed upon the circuit. This is true also, to a less extent, for values of current taken either side of the point of resonance. From this it may be seen that, at resonance, a dangerous voltage may occur across pieces of apparatus which are designed for lower voltages. Use is made of this in the building up of voltage in the case of synchronous converters, running at the end of a transmission line. In this case the synchronous converter is run over-excited to balance the inductance of the line.

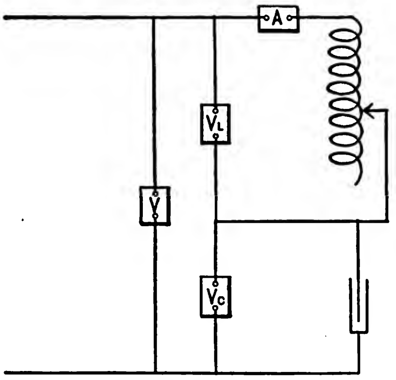

FIG. 5*A*. Connections for a series resonant circuit; resistance, capacity and emf. constant, variable inductance.

Suggestion. A convenient form of inductance for use here consists of a large double solenoid of heavy wire, the solenoid *containing no iron* and being so arranged that one part telescopes the other. By a proper distribution of the turns, the inductance can be made practically zero when the coils are telescoped and the current traverses the coils in opposite directions. It is a maximum when the coils are completely telescoped and the cur-

rent is in the same direction in both coils. A ready means for reversing the connections in one of the coils should be provided. By starting with the coils completely telescoped, a wide range of adjustment is possible by withdrawing the movable coil to the extreme limit of its travel, then reversing its connections and completely telescoping it.

Should the current pass through a maximum value without a condition of complete resonance, the cause may be sought in a variation of the pressure wave from the sine law; *i. e.,* the fundamental may be in resonance and the pressures across the inductance and the capacity may be unequal, due to the presence of one or more higher harmonics. This is a condition quite common in laboratory work. It can be minimized by the use of an alternator having a wave form closely approximating a sine wave and having a load capacity largely in excess of the demands of the experiment.

Data. The value of the capacity should be chosen so that it may be neutralized by the inductance, as shown by the above equation. The inductance should be varied from a low value to one considerably beyond that causing resonance. Readings of current are to be taken for the different values of L, and the pressure and frequency should be kept constant for each value of L. See Figure 5A.

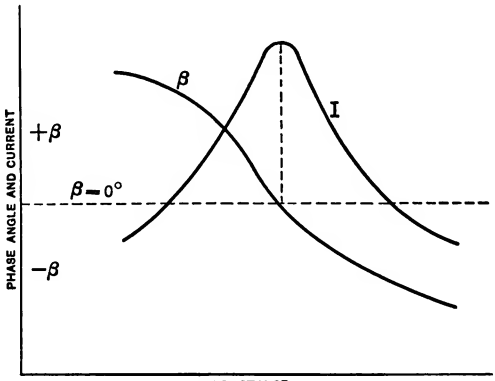

Fig. 5B. Curves showing the effect of variable inductance in a series resonant circuit.

Curves. Curves should be plotted showing the variation of phase angle and current with each change in L. Plot values of L as abscissas and values of current and phase angle as ordinates, as in Figure 5*B*. Curves should also be plotted showing the variation of pressure across the inductance. Indicate the point of resonance on each of the curves plotted. Check this point by computation.

Graphical Solution. Draw pressure diagrams for various values of inductance.

No. 6. VALUE OF CURRENT AND PHASE ANGLE IN A CIRCUIT OF VARYING CAPACITY; THE RESISTANCE, INDUCTANCE, PRESSURE AND FREQUENCY REMAINING CONSTANT.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8, Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2^4, pp. 24 to 79.

Object. To vary the value of the capacity in a series circuit containing constant resistance and inductance, operated at constant frequency and pressure, and to note the change in the values of the current and phase angle.

Theory and Method. The general theory is similar to that given in Experiment 5. Here the capacity is varied instead of the inductance. Resonance occurs when

$$C = \frac{1}{(2\pi f)^2 L}.$$

The current becomes a maximum and the phase angle zero at that point. For values of C greater than this, the angle β be-

comes positive; for smaller values β becomes negative. Connections should be made as shown in Figure 6*A*.

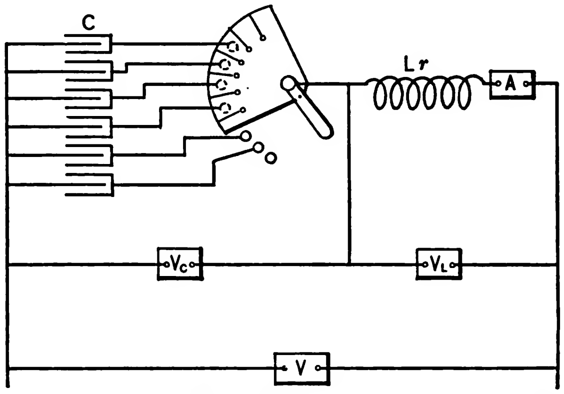

Fig. 6*A*. Connections for a series resonant circuit; resistance, inductance and emf. constant, variable capacity.

Suggestion. It is necessary to employ a considerable capacity. It is, therefore, desirable to have a number of condensers, each having a capacity of one or more microfarads, and also one or more subdivided condensers.

Care should be taken to discharge the condensers before making changes in the connections. Dangerous voltages may exist near resonance, and precautions should be taken to guard against accidents to persons and to condensers and other apparatus.

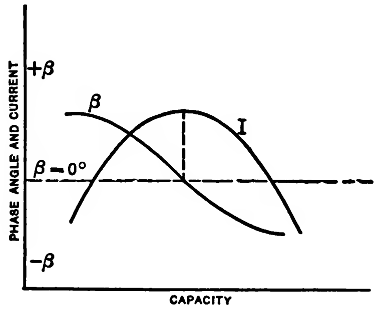

Fig. 6*B*. Curves showing the effect of variable capacity in a series resonant circuit.

Data. The capacity should be varied from a high value to one considerably below that necessary to cause resonance. Readings of the current should be taken for each value of capacity, the pressure and frequency being maintained constant.

Curves. Curves should be plotted showing the variation of phase angle and current with the changes in capacity. Plot values of C as abscissas and values of phase angle and current as ordinates. (See Figure 6*B*.) Curves should also be plotted showing the variation of pressure across the inductance, and capacity, with variations in capacity. Indicate the point of resonance on each of the curves plotted. Check this point by computation. The same comment as to the effect of wave form upon resonance in Experiment 5 applies here.

Graphical Solution. Draw pressure diagrams for various values of capacity. The phase angle for each value of capacity should be computed.

No. 7. VALUES OF CURRENT AND PHASE ANGLE IN A SERIES CIRCUIT OF VARYING FREQUENCY; THE RESISTANCE, INDUCTANCE, CAPACITY AND PRESSURE REMAINING CONSTANT.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8, Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2^4, pp. 24 to 79.

Object. To note the change in the value of the current and phase angle of a series circuit with a variation in the frequency.

Theory and Method. The general theory and method are similar to those outlined in Experiments 5 and 6. In the in-

vestigation of the phase angle, it is evident from the formula

$$\beta = \tan^{-1}\left[\frac{2\pi f L - \dfrac{1}{2\pi f C}}{R + r}\right],$$

that the phase angle becomes zero when

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

When f is less than this, β is negative, and the current leads the pressure. The current decreases rapidly for values of f which are greater or less.

In the case of transmission lines or other circuits containing inductance and capacity in various combinations, the chance for resonance becomes important. The line may be free from resonance at the normal frequency and with a sine wave impressed, but for the higher harmonics it may be in resonance. For this reason it may be important to know the shape of the pressure wave as well as its fundamental frequency.

Again, higher frequency surges may be set up by the opening of switches or circuit-breakers under load, or by lightning discharges which, with the combination of inductance and capacity present, may cause a transformer to break down or the line to discharge between the wires, followed by a short circuit. These high potentials are due to the presence of resonance for a given frequency and the consequent rise in potential across the inductance and capacity.

Data. The frequency f should be varied from a low value to one considerably greater than that which causes resonance. Readings of current are to be taken for the various values of f, the pressure being maintained constant.

Curves. Curves are to be plotted showing the variation of phase angle and current with variation of frequency; plot values of frequency as abscissas and values of current and β as ordinates.

Suggestion. The same comment applies here in reference to

resonance as in Experiments 5 and 6. As an extra precaution against wave form distortion in this case, the alternator should be operated at constant excitation of sufficient value to produce a stiff field and the constant pressure should be obtained by connecting the series circuit across a non-inductive resistance, potentiometer style, which is in turn connected across the terminals of the alternator. A little adjustment at the start will determine whether or not the experiment can be made over the desired range.

Graphical Solution. Draw pressure diagrams for various values of frequency. The phase angle for each value of f should be computed.

Question. At what value of frequency would resonance occur? Which odd harmonic of the commercial frequencies would be liable to produce resonance in this circuit?

No. 8. IMPEDANCE OF A CIRCUIT CONTAINING RESISTANCE AND INDUCTANCE IN PARALLEL.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8, Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2^4, pp. 24 to 79.

Object. To measure the impedance of a circuit containing a non-inductive and an inductive resistance in parallel, when an alternating pressure is impressed upon it.

Theory and Method. When a resistance and an inductance are connected in parallel, each will take a value of current depending upon its impedance and the pressure impressed upon it. The total current will be the vector sum of the current in both branches. This value will be less than the arithmetical sum of

the current in the several branches. If the circuits are connected as in Figure 8*A* and the necessary readings taken, diagrams of

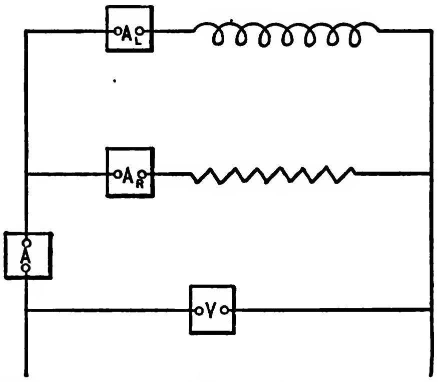

FIG. 8*A*. Connections for measuring the impedance of a circuit containing resistance and inductance in parallel.

currents may be drawn as in Figure 8*B*. Figure 8*B* may be drawn from the relation of the currents as read by the meters. The phase relations may be checked by determining the resistance and inductance of each branch, as in Experiment 1, from the voltmeter and ammeter readings. When the value of the angle β' is small, the determination of the proper relations becomes difficult from the construction. The value of β' may then be determined as in Experiment 1 and used in drawing the diagram.

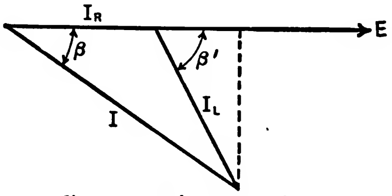

FIG. 8*B*. Current diagram, resistance and inductance in parallel.

Data. The apparatus used should be connected as in Figure 8*A* and readings of current and pressure taken as there indicated. Vary the resistance and take several sets of readings at constant pressure and frequency. Also obtain data necessary to calculate *r*.

Calculate. The value of the impedance and the phase angle of the circuit for the various values of R.

Graphical Solution. Construct the current diagram to scale, indicating the various components and angles with their values.

Curves. Plot curves showing the variation of the impedance and the total current with variation of the resistance R.

No. 9. IMPEDANCE OF A CIRCUIT CONTAINING NON-INDUCTIVE RESISTANCE AND CAPACITY IN PARALLEL.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8, Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2^4, pp. 24 to 79.

Object. To measure the impedance of a circuit containing non-inductive resistance and capacity in parallel, when an alternating pressure is impressed upon the circuit.

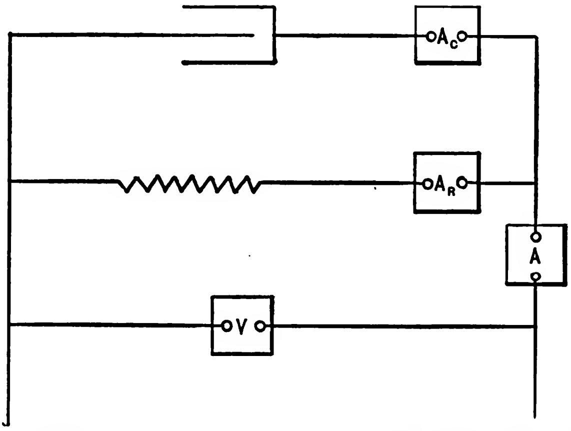

FIG. 9*A*. Connections for measuring the impedance of a circuit with resistance and capacity in parallel.

Theory and Method. When a resistance and a capacity are connected in parallel, the current through the resistance will be in phase with the pressure and that through the capacity in advance of the pressure. The current through each branch will equal the pressure divided by the impedance of that branch. The total current will be the vector sum of the currents in the two branches, and will be less than their arithmetical sum. The proper phase relations between these quantities may be found by diagrams as shown in Figure 9*B*. The current through the capacity will be nearly at right angles with the pressure axis.

Data. The connections should be made as shown in Figure 9*A* and readings of pressure and current taken as there indicated. Vary the resistance and take several sets of readings at constant pressure and frequency.

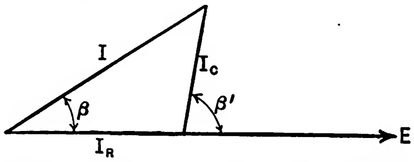

FIG. 9*B*. Current diagram, resistance and capacity in parallel.

Calculate. The value of the impedance and phase angle of the circuit for the different values of *R*.

Graphical Solution. Construct the current and admittance diagrams for the circuit, indicating the value of each vector and the angle between them.

Curves. Plot curves showing the variation of the impedance and current, with variation of the resistance *R*.

No. 10. IMPEDANCE OF CIRCUIT CONTAINING RESISTANCE, INDUCTANCE AND CAPACITY IN PARALLEL.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8,

Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4, Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2[4], pp. 24 to 79.

Object. To measure the impedance of a circuit containing resistance, inductance and capacity in parallel, when an alternating pressure is impressed upon the circuit.

Theory and Method. When the branches are in electrical resonance, the current in the external circuit is a minimum, while the current between the inductance and capacity may be many times as large as the current in the main circuit. In a parallel resonant circuit containing no resistance or hysteresis effect, there would be no losses in either branch. In this case the current in the external circuit would be zero, while that being transferred between condenser and inductance would be

$$I = \frac{E}{2\pi f L} = 2\pi f C E.$$

This current would flow, if the pressure E were impressed upon either branch alone.

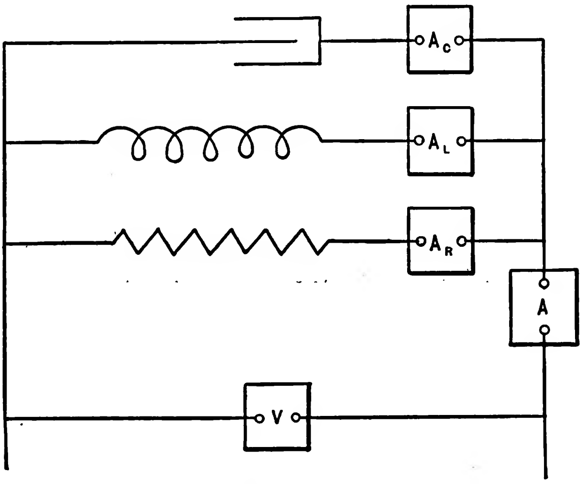

Fig. 10*A*. Connections for measuring the impedance of a circuit containing resistance, inductance and capacity in parallel.

Data. The apparatus used should be connected as shown in Figure 10*A* and readings of current and pressure taken as there indicated. Vary the resistance and take several sets of readings. Also obtain data necessary to calculate r. Take a set of readings with the non-inductive resistance circuit eliminated; this may be done by opening the circuit through ammeter A_1 in Figure 10*A*.

Calculate. The values of the impedance, admittance, susceptance and phase angle of the circuit for the different values of R.

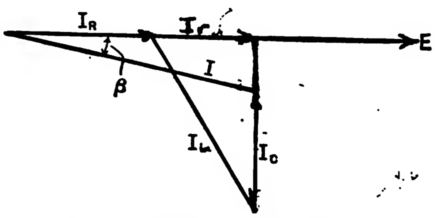

FIG. 10*B*. Current diagram, resistance, inductance and capacity in parallel.

Graphical Solution. Construct the current and admittance diagrams for the circuit, indicating the value of each vector and its phase angle with respect to the impressed pressure.

Figure 10*B* may be laid off from the readings of the ammeters and the angles determined by measurement. In drawing the diagrams, care should be taken to remember that the current through an inductance lags behind and that through a capacity leads, the impressed pressure.

Question. With a state of resonance and low resistance leads between capacity and inductance in this experiment, what would be the value of the circulating current between these two branches? What would be the total current?

No. 11. IMPEDANCE OF A CIRCUIT CONTAINING RESISTANCE, INDUCTANCE AND CAPACITY IN VARIOUS COMBINATIONS OF SERIES AND PARALLEL CONNECTION.

References. Bedell and Crehore; Franklin and Esty, Chap. 1 to 5; Steinmetz' "Elements," Part 1, Sec. 7 to 17; Karapetoff, Chap. 5 and 6; Thomälen, Chap. 4; Stewart and Gee, Chap. 8, Art. 167; Carhart and Patterson, Chap. 7; Arnold, Vol. 1, Chap. 2 to 4; Steinmetz' "A.C. Phenomena," Chap. 5, 8 and 9; Thompson's "Dynamos," Vol. 2, Chap. 1; Handbuch der Elektrotech., Vol. 2^4, pp. 24 to 79.

Object. To consider various combinations of the circuits taken up in the preceding experiments.

Theory. When resistance, inductance and capacity are combined in series and parallel, it is difficult to obtain a simple equation for current, such as has been used heretofore. If, however, the equivalent impedance of each of the divided circuits be obtained, the general formula given in Experiment 4 may be used to obtain the relations between current, pressure across the various parts and the various phase angles. Then the branch circuits may be treated independently in a manner similar to that used in Experiments 8, 9 and 10.

Data. One or more combinations should be tried in which suitable readings are obtained. All data should be graphically represented.

Suggestion. It is desirable in this experiment to approximate the conditions of an actual transmission line with different loads, to show the determination of line drop.

If preferred, Experiments 12, 13, 14 and 15 may be substituted for this experiment.

No. 12. REGULATION OF A TRANSMISSION LINE CONTAINING NEGLIGIBLE INDUCTANCE AND CAPACITY.

References. Franklin and Esty, Chap. 15; Arnold, Vol. 1, Chap. 21; Karapetoff, Chap. 13; Steinmetz' "A.C. Phenomena," Chap. 11 and 12; *Elec. Jour.*, December, 1905, C. F. Scott, Transmission Circuits, February, 1906, C. F. Scott, Induction in transmission circuits; *Elec. Wld. and Eng.*, May 18, 1901, F. G. Baum, Regulation diagrams; *Elec. Jour.*, June, 1905, J. S. Peck, How to calculate regulation of alternating current circuits; Steinmetz' "Elements," Part 1, Art. 12.

Object. Test of a transmission line containing resistance, but negligible inductance and capacity, under different conditions of loading.

Theory and Method. The impedance drop in this type of line depends upon the resistance and current, or

$$E = IR.$$

The "line drop" is the numerical difference between the potentials at the two ends of the line. This is equal to the impedance drop when the load is non-inductive.

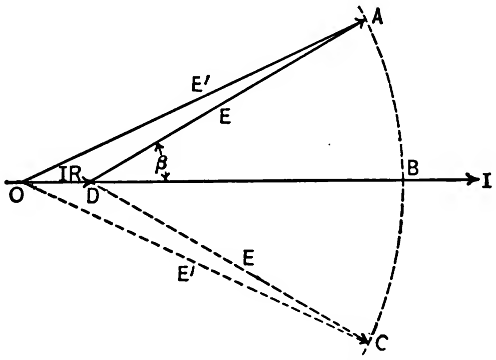

FIG. 12. Electromotive force diagram, transmission line containing negligible inductance and capacity.

If values of R and current be chosen to represent similar

conditions on a higher voltage line, a value of E' may be chosen which will suit the ordinary laboratory apparatus. The diagram will be similar to that for higher voltage drawn to a scale giving the same length of vectors. In Figure 12 the triangle *ADO* is taken for an inductive load at the receiving end and *CDO* for capacity load taking the same current. The diagram is drawn for constant pressure at the sending end. A similar diagram could be drawn for constant pressure at the receiving end of the line. In Figure 12,

$E' =$ the voltage at the sending end,
$E =$ the voltage at the receiving end,
$R =$ the resistance of the line, and
$\cos \beta =$ the power factor of the load at the receiving end.

Data. Connect a 110-volt alternating current source of supply to a non-inductive load, through a low non-inductive resistance. Take readings, with varying loads and constant impressed voltage, of resistance drop and voltage across the load. Repeat with inductive and capacity loads of constant power factor.

Compute. The line drop, resistance drop and percent. regulation, for each condition.

Curves. Plot curves with load current as abscissas and with voltage across load and line drop as ordinates, for each condition of loading. Draw pressure diagrams illustrating these various conditions.

No. 13. REGULATION OF A TRANSMISSION LINE CONTAINING RESISTANCE AND INDUCTANCE BUT NEGLIGIBLE CAPACITY.

References. Franklin and Esty, Chap. 15; Arnold, Vol. 1, Chap. 21; Karapetoff, Chap. 13; Steinmetz' "A.C. Phenomena," Chap. 11 and 12; *Elec. Jour.*, December, 1905, C. F. Scott, Transmission circuits, February, 1906, C. F. Scott, Induction in transmission circuits; *Elec. Wld. and Eng.*, May 18, 1901, F. G, Baum,

Regulation diagrams; *Elec. Jour.*, June, 1905, J. S. Peck, How to calculate regulation of alternating current circuits; Steinmetz' "Elements," Part 1, Art. 15 and 16.

Object. To study the regulation of a transmission line having resistance and inductance, but negligible capacity.

Theory and Method. As in the case of a line having resistance only, the regulation depends upon the character of the load as well as upon the current flowing. It is the purpose here to investigate a given line under different conditions of loading, in order to ascertain its characteristics.

In Figure 13, if

E' = the voltage at the generator,
E_1, E_2, E_3 = the voltages at the receiving end,
R = the line resistance,
X = the line reactance, and
$\cos\beta$ = the power factor of load,

the relation between these quantities may be clearly seen from the diagram, when the value of the load current and the generator voltage remain constant.

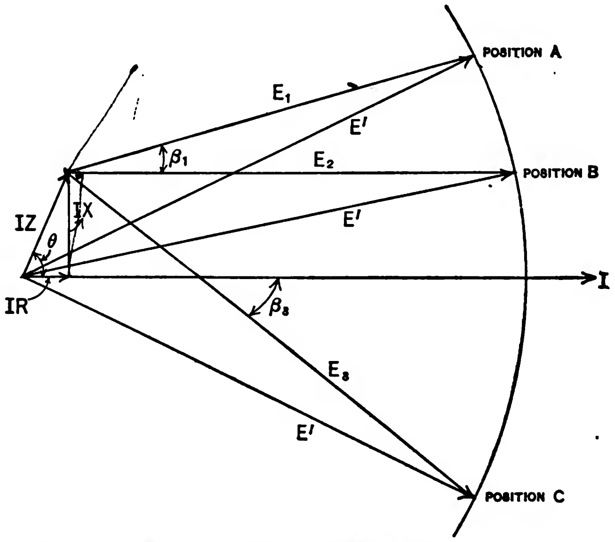

Fig. 13. Electromotive force diagram, transmission line containing resistance and inductance but negligible capacity.

Here Position A shows an inductive load, Position B a non-inductive load and Position C a capacity load, at the receiving end. When the value of X is fairly large, the receiving voltage may be kept constant or made to increase so that it becomes greater than the sending voltage, for certain values of β with capacity load. Advantage is taken of this fact to keep the voltage of the direct current side of a synchronous converter constant with varying loads, by over-exciting the converter. When the transmission line reactance is not enough to produce this effect, a reactance is also located at the terminals of the synchronous converter. It will be seen that Figure 13 is drawn by referring all conditions to current as a base line. The IR and IX drops are laid off respectively in phase with and at right angle to the current. The origin is taken as the center of a circle with radius equal to E'. The positions and values of E are found by laying off a line from the end of the impedance drop line at the proper angle with respect to I, representing the power factor of the load. The value of the angle θ is shown by the formula

$$\theta = \tan^{-1}\frac{X}{R},$$

and it is assumed that this angle is constant for all loads.

Data. Take a set of inductances and resistances and approximate as nearly as possible to a transmission line, with constants reduced to the basis of a 110-volt circuit. Pass current through this from a 110-volt alternating current source to a non-inductive load which may be varied. Take readings of impressed voltage, receiving voltage and current for the various loads. Repeat with inductive and capacity loads of constant angle of lag or lead.

Compute. The line drop, impedance drop in the line and per-cent. regulation, for each condition of loading.

Curves. Plot curves with load current as abscissas and with voltage across load and line drop as ordinates, for each condition of loading. Draw representative diagrams similar to Figure 13.

No. 14. REGULATION OF A TRANSMISSION LINE CONTAINING RESISTANCE AND CAPACITY BUT NEGLIGIBLE INDUCTANCE.

References. Franklin and Esty, Chap. 15; Arnold, Vol. 1, Chap. 21; Karapetoff, Chap. 13; Steinmetz' "A.C. Phenomena," Chap. 11 and 12; *Elec. Jour.*, December, 1905, C. F. Scott, Transmission circuits, February, 1906, C. F. Scott, Induction in transmission circuits, June, 1905, J. S. Peck, How to calculate regulation of alternating current circuits; *Elec. Wld. and Eng.*, May 18, 1901, F. G. Baum, Regulation diagrams; Steinmetz' "Elements," Part 1, Art. 15 and 16.

Object. To investigate, for various types of loads, the regulation of a transmission line having resistance and capacity, but negligible inductance.

Theory and Method. A transmission system consisting of cables may have negligible inductance, but appreciable capacity and resistance. In this case a considerable charging current would be required from the generating station. This may equal full load current of one or more generators, although it represents little power consumption since the power factor is nearly zero.

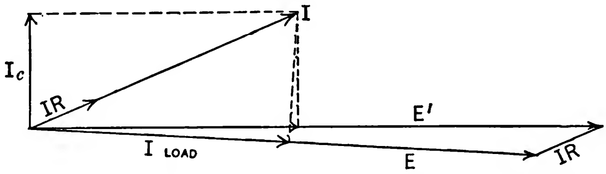

FIG. 14*A*. Electromotive force diagram, transmission line containing resistance and capacity but negligible inductance.

When load is added at the receiving end, its effect may not be noticeable on the station ammeters, particularly if the load is slightly inductive. It would, of course, appear on the wattmeters. If the line takes a large charging current, an inductive load at the receiving end may even lower the reading of the station ammeters. This may be seen in Figures 14*A* and 14*B*. Here the *IR* drop is in phase with the load current at the receiving end.

These diagrams are drawn considering the capacity of the cables as concentrated at the sending end.

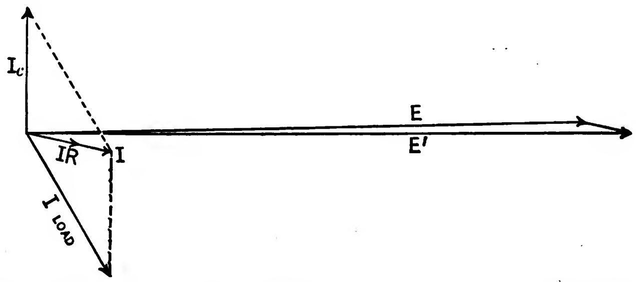

FIG. 14*B*. Electromotive force diagram, transmission line containing resistance and capacity but negligible inductance.

Data. Arrange a set of condensers to represent a transmission line with capacity and resistance. Place a load across the line in parallel with the condensers. Starting with a non-inductive load, take readings to enable diagrams to be plotted from no load to full load. Load with inductive load of constant power factor and take a similar set of readings. Take a similar set with capacity loads.

Suggestion. If a sufficient number of condensers is not available for use at low voltage, a transformer may be employed. In this case the condensers are connected across the high voltage coil and the low pressure coil is connected across the line. Care should be taken that the transformer potential is not too high for the condensers.

Compute. The line drop, impedance drop and percent. regulation, for each condition of loading.

Curves. Plot curves between load current and total current, and load current and angle of lag or lead at the sending end of the line, for all conditions of loading, using load current as abscissas.

No. 15. REGULATION OF A TRANSMISSION LINE HAVING CAPACITY, INDUCTANCE AND RESISTANCE.

References. Franklin and Esty, Chap. 15; Arnold, Vol. 1, Chap. 21; Karapetoff, Chap. 13; Steinmetz' "A.C. Phenomena," Chap. 11 and 12; *Elec. Jour.*, December, 1905, C. F. Scott, Transmission circuits, February, 1906, C. F. Scott, Induction in transmission circuits, June, 1905, J. S. Peck, How to calculate regulation of alternating current circuits; *Elec. Wld. and Eng.*, May 18, 1901, F. G. Baum, Regulation diagrams; Steinmetz' "Elements," Part 1, Art. 15 and 16.

Object. To study the regulation of a transmission line having capacity, inductance and resistance.

Theory and Method. The long distance transmission of power requires lines to be used whose capacity, inductance or resistance cannot be neglected. Such a line has a distributed capacity; that is, the current and electromotive force have different phase relations at different points on the line. The operation of such a line may be studied by means of approximations, according to the condition to be studied.

When the charging current for the line is small compared with the load current, the capacity may be considered as concentrated

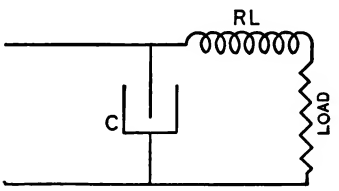

FIG. 15*A*. Connections for transmission line tests with capacity at the generating end.

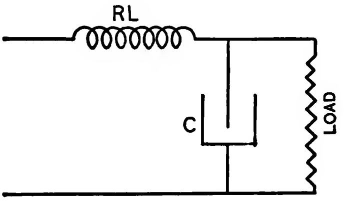

FIG. 15*B*. Connections for transmission line test with capacity at the receiving end.

in one condenser placed at either end or in the middle of the line. The inductance and resistance are also concentrated according to the position of the condenser, as in Figure 15*A*, 15*B* and 15*C*. Loads of different characters and values may be

connected to the receiving end, and the phase relations at different portions studied.

A closer approximation may be made by connecting condensers of one-sixth of the line capacity at each end and one of two-thirds the line capacity in the middle, as shown in Figure 15*D*.

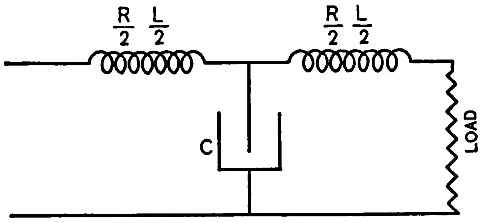

FIG. 15*C*. Connections for transmission line test with capacity at the middle of the line.

Data. Assume constants for some definite transmission line. Arrange connections by means of condensers, inductances and resistances, as shown in Figure 15*A*, 15*B*, 15*C* or 15*D*. Find the value and phase relation of the line current at different points on the line, with no load at the receiving end. Place a non-inductive

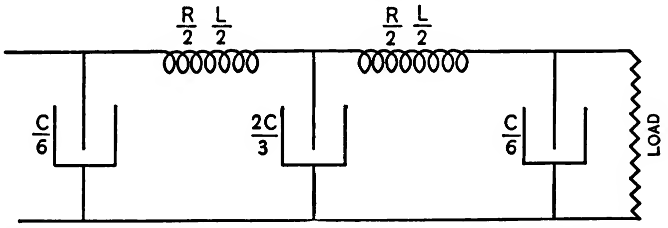

FIG. 15*D*. Connections for transmission line test with distributed capacity.

load at the receiving end and study the line as before. Repeat, using inductive and capacity loads of constant angle of lag or lead. Try some of the other forms of connections.

Compute. The line drop, impedance drop and percent. regulation, for each condition of loading.

Question. In the case of a long transmission line, explain how it would be possible for the ammeter reading at the sending end to decrease as the load was applied to the receiving end.

Curves. Plot curves as in Experiment 14.

INTRODUCTION TO POWER MEASUREMENTS.

In the performance of the following experiments, some consideration of power and its measurement in an alternating current circuit will be necessary.

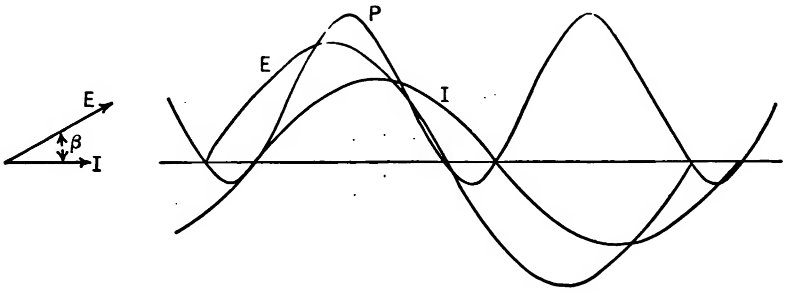

Fig. P1. Current, pressure and power waves, when the current lags behind the pressure by the angle β.

In a direct current circuit, power is represented by the expression

$$P = EI,$$

where E is the voltage drop and I is the current in the circuit.

As has been seen in the preceding experiments, the electromotive force in an alternating current circuit is seldom in phase with the current. If the current lags behind the electromotive force by the angle β, the electromotive force and current waves will be as shown in Figure $P1$. If the instantaneous values of current and electromotive force are multiplied together, there will result the power curve P in Figure $P1$. It will be seen that there are some positive and some negative loops in the power curve. The greater the angle β, the greater the value of the negative loops in comparison with the positive loops until, when β becomes 90 degrees, the positive and negative loops are alike and the value of power for the circuit is zero as shown in Figure $P2$.

When the current and electromotive force are in phase with each other, the loops are all positive (or, in the case of motor action with reversed direction of current vector, all negative) as seen in Figure $P3$. If the same values of current and electro-

[redacted]

motive force are taken in each case, it can readily be seen that the power in an alternating current circuit cannot be equal to EI in all cases. This may be expressed by the equation

$$P = EIp, \tag{1}$$

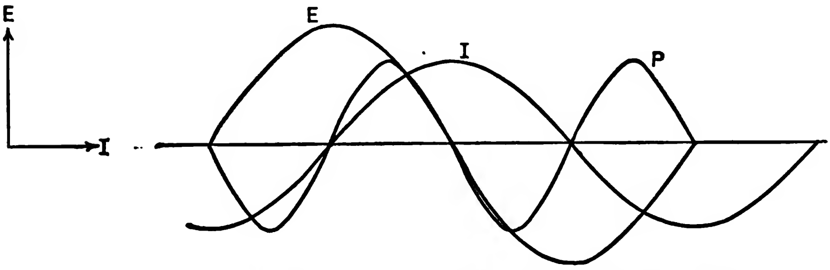

FIG. P2. Current, pressure and power waves, when the current lags 90 degrees behind the pressure.

where p is a quantity not greater than 1, which is called the *power factor* of the circuit. This may also be written

$$P = EI \cos \beta, \tag{2}$$

where $\cos \beta = p$, is the power factor. In the case of sine waves β is the angle of lag or lead between the current and electromotive force, as found by different methods in the preceding experiments.

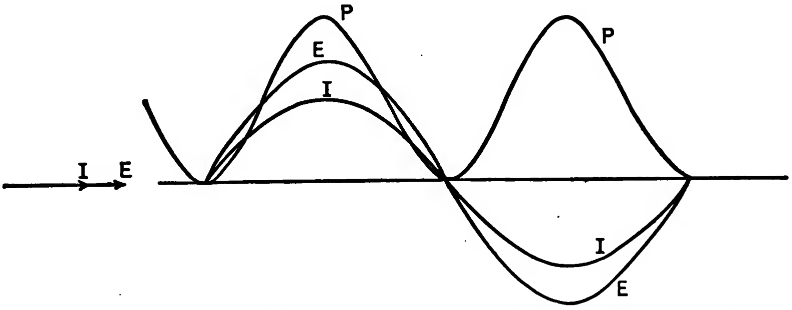

FIG. P3. Current, pressure and power waves, when the current and pressure are in phase.

Power in an alternating current circuit may be measured in numerous ways, as shown in the following experiments. The most convenient and common method is by means of a wattmeter.

From previous considerations of alternating current circuits,

$$E = I\sqrt{R^2 + (2\pi f L)^2}, \tag{3}$$

for a circuit containing resistance and inductance. Multiplying by cos β,

$$E \cos \beta = I(\sqrt{R^2 + X^2}) \cos \beta$$
$$= RI,$$

or

$$EI \cos \beta = RI^2.$$

Since

$$P = EI \cos \beta,$$
$$P = RI^2. \tag{4}$$

That is, all the power in a coil without an iron core, is expended in heating the coils due to their resistance, as in a direct current circuit.

When iron is placed in the core of the coil or the circuit contains some power-absorbing device other than resistance, this equation does not hold where R is the resistance of the circuit. Dividing Equation 4 through by I^2, the following equation is derived:—

$$R = \frac{P}{I^2}.$$

The value of R as obtained from this expression, is sometimes called the equivalent or effective resistance of the circuit. It will be seen that this is not a constant quantity for the circuit, except under certain restricted conditions. It varies with the load applied, with the frequency of the circuit in some cases and with the value of the magnetization in the iron core. It is useful in the consideration of certain conditions of a transformer or other coil containing iron in its core.

No. 16. MEASUREMENT OF POWER BY THE THREE VOLTMETER METHOD.

References. Lamb, p. 38; Fleming, Vol. 1, p. 488; Gray, Vol. 2, pp. 694 and 695; Karapetoff, p. 131; Russell, Vol. 1, p. 204; Standard Handbook, Sec. 3, Art. 260 to 269; Handbuch der Elektrotech., Vol. 2^4, pp. 130 to 134; Arnold, Vol. 1, p. 205.

Object. The measurement of power expended in an alternating current circuit, by means of three voltmeters.

Theory and Method. In all measurements of power in an alternating current circuit, it is necessary to take into consideration the fact that the current may not be in phase with the impressed pressure. The general formula for power is

$$P = IE \cos \beta,$$

where β is the angle of lag or lead. If we have some means of determining the value of $\cos \beta$, the power of a circuit is easily obtained.

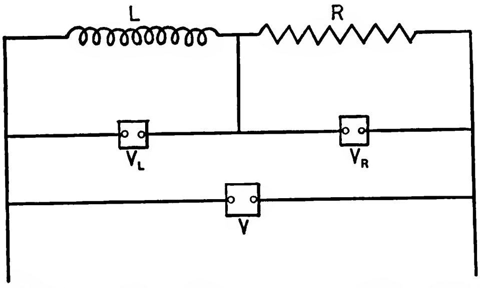

FIG. 16*A*. Connections for the measurement of power by the three voltmeter method.

In Experiment 1, a method was used for determining the lag angle in a series circuit. The angle between the current and pressure in each part of the circuit was also determined. If a known non-inductive resistance is placed in series with the circuit whose power is to be measured, Figure 16*A*, and readings taken as shown, a triangle of pressures may be constructed as in Figure 16*B*. The current in the circuit must be in phase with E_R and

hence may be represented by

$$OI = \frac{E_R}{R} = I.$$

Also, the current lags behind E_L by the angle β. From the triangle, OAB, we obtain

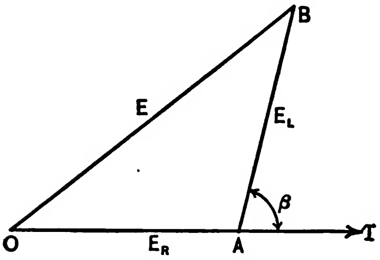

FIG. 16*B*. Pressure diagram for the measurement of power by the three voltmeter method.

$$E^2 = E_R^2 + E_L^2 - 2E_R E_L \cos OAB,$$

or

$$E^2 = E_R^2 + E_L^2 + 2E_R E_L \cos \beta.$$

From which

$$\cos \beta = \frac{E^2 - E_R^2 - E_L^2}{2E_R E_L}.$$

Finally, the power absorbed in the inductive circuit is

$$P_L = E_L I \cos \beta = \frac{I}{2E_R}(E^2 - E_R^2 - E_L^2) = \frac{1}{2R}(E^2 - E_R^2 - E_L^2).$$

The results should be represented by means of diagrams.

Suggestions. The accuracy of the readings may be checked by substituting a non-inductive resistance for the inductive resistance.

It may be seen from Figure 16*B* that, in order to obtain the greatest degree of accuracy, E_R and E_L should be equal or nearly so. In measuring the power supplied to a transformer, a pressure equal to the normal one for which the transformer is designed, should be impressed across the terminals. The resultant pressure across the two parts of the circuit must thus be nearly twice the normal pressure of the transformer.

All the voltmeter readings enter into the result as squares, so an error in reading the instrument is magnified in the final result. Notwithstanding the disadvantages of this method, it was, until the general introduction of the wattmeter, one of the

most common methods of measuring power in an alternating current circuit.

A disadvantage of this method is that, for greatest accuracy, the power absorbed is in most cases more than twice that of the measured circuit. Also, there must be available a source of pressure greater than that of the circuit to be measured. Due to a distortion of the pressure wave and the resulting flat top flux wave, the losses in a transformer are less with the same effective voltage across its terminals when resistance is included in circuit with it. A considerable error may result in obtaining iron losses.

It is often more convenient to use an ammeter in the circuit when the resistance R is unknown or is dependent on the current.

Data. Measure the power absorbed by an inductive resistance, such as the low pressure winding of a transformer on open circuit, using the three voltmeter method. If a transformer is used, let the pressure on its terminals be normal. It is advisable to carefully separate and tape over the terminals of the high tension winding. Measure the power with a wattmeter, as a check.

Diagram. Draw a triangle of pressures and show the angle of lag in the inductive portion of the circuit.

Calculate. The power expended in the inductive load, from the three pressure readings, and compare the result with the wattmeter reading.

No. 17. MODIFICATION OF THE THREE VOLTMETER METHOD OF MEASUREMENT OF POWER.

References. Lamb, p. 39; Russell, Vol. 1, p. 207; Handbuch der Elektrotech., Vol. 2^4, p. 134; Standard Hankbook, Sec. 3, Art. 260 to 269; Arnold, Vol. 1, pp. 199 to 209; Fleming, Vol. 1, Chap. 5.

Object. To overcome some of the objections to the three voltmeter method of measuring power.

Theory and Method. It has been shown in Experiment 16 that two very decided objections to the use of the three voltmeter method of measuring power, are the large amount of power taken in the non-inductive resistance and the comparatively large terminal voltage necessary for the condition of maximum accuracy. To overcome these objections, Professor Ryan has used an ingenious modification by which the extra pressure is cut down to a small percentage of the pressure across the inductive resistance, while still maintaining the conditions for greatest accuracy.

In the three voltmeter method of measuring power the three voltmeter readings are taken simply to find the value of the cosine of the angle of lag or the power factor. This value, as developed in Experiment 16, is

$$\cos\beta = \frac{E^2 - E_R^2 - E_L^2}{2E_R E_L}.$$

In Figure 17*A* the connections of Figure 16*A* are changed so as to accomplish the same end by using a smaller resistance R

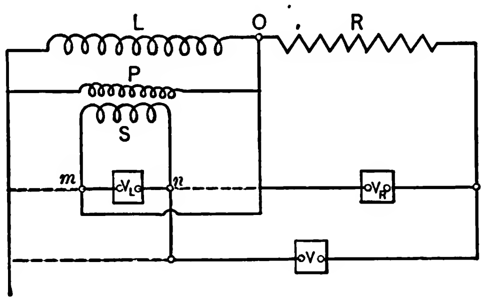

FIG. 17*A*. Connections for the measurement of power by the modified three voltmeter method.

and yet attain the same degree of accuracy (the dotted lines indicate the connections used in Experiment 16). A potential transformer PS is placed across the inductive circuit L, the secondary of which is connected to the voltmeter V_L, thus reducing the reading E_L in the ratio of the potential transformer. Since this transformer practically is on open circuit (the voltmeter current being the only load) the primary and secondary pressures are in

direct opposition, and, in order to add the reduced pressure across L to that across R, it is necessary to connect terminal m to point o, and connect the voltemeter V to terminal n of the secondary.

Referring to Figure 17B, triangle OAB represents the vector diagram as obtained in Experiment 16. It is now evident that, if R be unchanged, the new diagram will be OAC, AB/AC representing the ratio of the potential transformer. The angle β, however, is the same as before. If, now, R is reduced in the same ratio, and readings taken as before, the triangle will be ACD, similar to OAB. However, this is not necessary, as any reduction of R that will make AD nearly equal to AC will be satisfactory, since β will remain the same no matter what the value of R. It must be borne in mind that the readings E_L' and E' are not the voltages across L and the total circuit as in Experiment 16. Denoting the voltages in this case by E_L', E_R' and E', the angle β may be obtained from the equation

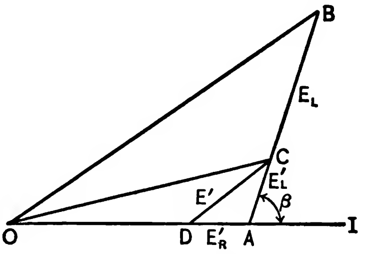

FIG. 17B. Pressure diagram for the measurement of power by the modified three voltmeter method.

$$\cos\beta = \frac{E'^2 - E_R'^2 - E_L'^2}{2E_R'E_L'}.$$

The power taken by the inductive circuit is

$$\text{watts} = IE_L\cos\beta = IE_L\frac{E'^2 - E_R'^2 - E_L'^2}{2E_R'E_L'}.$$

where E_L is the actual pressure across L. But

$$\frac{E_L}{E_L'} = K,$$

the ratio of the transformer, hence

$$\text{watts} = KI\frac{E'^2 - E_R'^2 - E_L'^2}{2E_R'}.$$

If the resistance R is known, this formula may be expressed as follows:—

$$\text{watts} = K\frac{E'^2 - E_R'^2 - E_L'^2}{2R},$$

where R is the non-inductive resistance.

The power taken by the transformer may be ascertained by taking readings with the inductive load cut out. The advantage of this method is that the resistance A and the attendant loss can be made much smaller than if the auxiliary transformer were not used.

Data. Measure the power absorbed by an inductive resistance upon which a high pressure is impressed, such as the high tension winding of a transformer; also obtain data necessary to calculate R.

Diagram. Draw a triangle of pressures and show the angle of lag in the inductive load.

Calculate. The power expended in the inductive load from the three pressure readings and compare the results with wattmeter readings.

No. 18. MEASUREMENT OF POWER BY THE THREE AMMETER METHOD.

References. Lamb, p. 39; Fleming, Vol. 1, p. 492; Russell, Vol. 2 p. 206; Handbuch der Elektrotech., Vol. 2^4, p. 137; Standard Handbook, Sec. 3, Art. 260 to 269; Arnold, Vol. 1, pp. 206 to 208.

Object. One of the principal objections to the three voltmeter method of measuring power was shown to be the necessity of a pressure in excess of the normal pressure of the circuit to be tested. The three ammeter method is somewhat similar but does not require a source of pressure higher than that which is ordinarily used on the circuit tested. But, on the other hand, it takes nearly twice the normal current.

Theory and Method. A known non-inductive resistance is placed in parallel with the circuit tested and ammeter readings are taken as indicated in Figure 18*A*, for the purpose of deter-

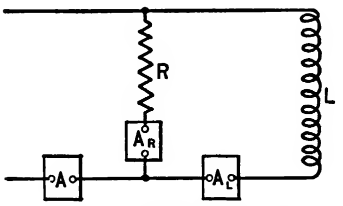

FIG. 18*A*. Connections for the measurement of power by the three ammeter method.

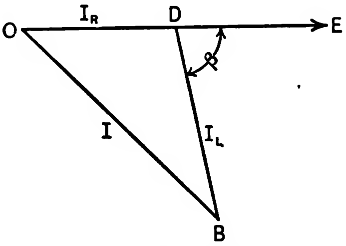

FIG. 18*B*. Current diagram for the measurement of power by the three ammeter method.

mining the phase difference between current and pressure in the inductive circuit.

Since the vector sum of I_R and I_L must equal I, these three values of current must have the relations shown in Figure 18*B*. The pressure across the resistance must be in phase with I, and may be represented by OE. In the triangle OBD we have the relation

$$I^2 = I_R^2 + I_L^2 + 2I_R I_L \cos\beta,$$

whence

$$\cos\beta = \frac{I^2 - I_R^2 - I_L^2}{2I_R I_L}.$$

Therefore, the power absorbed by the inductive circuit equals

$$P_L = I_L E \cos\beta = \frac{E}{2I_R}(I^2 - I_R^2 - I_L^2),$$

and finally,

$$P_L = \frac{R}{2}(I^2 - I_R^2 - I_L^2),$$

where R is the non-inductive resistance.

It is seen that, to obtain the greatest degree of accuracy, I_R must be equal to I_L and that the ammeter readings enter in the

form of squares so that great care should be taken in measuring the current.

A combination method is often employed in which a voltmeter is placed across the non-inductive resistance where the value of this resistance is not known. Lamps are convenient to use for this non-inductive resistance, but their resistance changes with the amount of current and hence the combination method must be used where lamps are employed.

Data. Measure the power absorbed by an inductive resistance, using the three ammeter method. If a transformer is used, let the pressure on its terminals be normal. Measure the power by a wattmeter, as a check. Obtain data necessary for calculating R.

Diagram. Draw a triangle of currents and show the angle of lag in the inductive circuit.

Calculate. The power expended in the inductive circuit from the three ammeter readings and compare the results with the wattmeter readings.

No. 19. MEASUREMENT OF POWER BY MEANS OF A WATTMETER.

References. Lamb, p. 34; Karapetoff, Chap. 4; Franklin and Esty, p. 47; Esty, pp. 66–68; Handbuch der Elektrotech., Vol. 2^4, p. 124; Swenson and Frankenfield, Vol. 1, p. 14.

Object. To study the use of a wattmeter for the measurement of power.

Theory and Method. Power in an alternating current circuit may be represented by

$$P = EI \cos \beta,$$

where β is the lag angle between the current and pressure in the circuit. The most convenient and, usually, the most accurate method of measuring this power, is by means of a wattmeter. The most common type of wattmeter for this purpose works

upon the electro-dynamometer principle. The current I is passed through a stationary coil and the pressure is impressed upon a circuit containing the movable coil and a resistance in series with it. The deflection is then indicated upon a scale, by a pointer attached to the movable coil.

Such an instrument is accurate enough for the ordinary measurements. An error in the true power may creep in through one of three causes. First, the inductance of the system, including the movable coil. This error is taken up and discussed in Experiment 22. Second, the loss of power in the current coil of the wattmeter being charged to the circuit, the power of which is being measured. Third, the loss of power in the movable coil circuit. The second and third may, of course, be determined by knowing the resistance and current in each path and subtracting the I^2R loss in the instrument from the power as indicated by the instrument. This would cause extra trouble and lead to inaccuracy.

The errors may be compensated by connecting the pressure coil of the meter so that its current also flows through the current coil and then passing this pressure coil current through a fixed fine wire coil wound concentric with the current coil and having the same number of turns, but magnetically opposed to it. Care must be taken, if this method of compensation is used, not to have too high a potential between the two stationary windings. This may be done by connecting the side of the pressure coil circuit containing the compensating coil to the side of the line passing through the current coil. This grounds the two coils together, preventing a high potential between the windings. This method should also be used when measuring power from circuits having high potentials, in order to avoid errors due to static charges.

Wattmeters operating on other principles are also in use and give very good satisfaction. Each has its own peculiar errors and these are taken up and investigated in later experiments.

Data. A non-inductive load should first be used and the power absorbed by it measured by the wattmeter. This should

be checked against the power computed from voltmeter and ammeter readings.

Measure the power absorbed by an air core inductance and also that absorbed by an iron core inductance. Measure each resistance by means of a direct current.

Compute. The value of the resistance of each circuit measured, by the equation

$$R = \frac{P}{I^2}.$$

Compare this value with that obtained from measurement with direct current instruments. Explain any difference in these values.

Discuss. In the report, discuss the wattmeter used in this experiment and its individual errors.

No. 20. EXPERIMENTAL DETERMINATION OF THE EFFECT OF FREQUENCY ON THE READINGS OF A VOLTMETER.

References. Gray, pp. 676 to 680; Handbuch der Elektrotech., Vol. 2^4, pp. 224 to 226; Vol. 2^5, pp. 46 and 47; *Trans. Am. Inst. Elec. Eng.,* June, 1905, E. F. Northrup, A new instrument for the measurement of alternating currents; *Elect'n Lond.,* November 25, 1904, W. E. Sumpner, The use of iron in alternating current instruments.

Object. To determine the effect of frequency on the indications of a voltmeter.

Theory and Method. All alternating current voltmeters which have appreciable self inductance in their circuits have their constants affected by any variation of frequency from that for which they were calibrated. Nearly all instruments of this class depend upon electro-dynamic action for their indications.

From the equation

$$I = \frac{E}{\sqrt{R^2 + (2\pi f L)^2}},$$

it will be seen that the current in a circuit depends upon the value of the frequency. Unless the value of $2\pi fL$ is small in comparison with R, the accuracy of the instrument will be affected. Furthermore it will be seen that, for the same value of pressure, the indications will be less, the higher the frequency. In the ordinary form of voltmeter the value of R is large in comparison with that of $2\pi fL$. The same torque is required for the moving systems of all voltmeters of the same type. In order to produce this torque it is customary to design voltmeters to have the same number of turns and to carry the same current. It will be seen that the self induction will be the same for nearly all instruments of the same type. Hence, the lower the voltage to be measured, the lower the value of the resistance which is put in series with the coils to limit the current. Therefore, low voltage instruments will be affected by frequency variation to a greater extent than similar higher voltage instruments.

To determine the effect of frequency it is necessary to compare the voltmeter with some form of pressure indicator which is not affected by the frequency, such as a hot wire or an electrostatic voltmeter. The comparison should be made, using a direct current and also alternating currents of two or more frequencies. In calibrating a dynamometer type instrument on direct current, the current should be reversed and two readings taken for each value of current. The mean of the two readings should be used. The reason for differences in the readings may be due to the fact that the coils are not located centrally with respect to each other or that the reading may be affected by a stray magnetic field. With alternating current the rapid reversal of current corrects the error. Although preferable, it is not necessary that the reference voltmeter be a standard one. Make the comparisons both on increasing and on decreasing values of pressures, in order to detect any error due to friction in the instrument bearings. In ascending, always approach the desired value from a lower one and, similarly, in descending approach this value from a higher one.

Suggestion. If masses of metal have been used in the construction of the voltmeter tested, the indications for alternating currents, besides being affected by the inductance in the windings, are affected by the dynamic effect of the eddy currents set up in these masses. This effect cannot be calculated with any degree of accuracy, but in modern instruments may generally be neglected.

Data. Compare the two voltmeters for both ascending and descending values of pressure, using both direct and alternating pressures. Use two or more frequencies if possible.

Curves. Plot curves (see Figure 20) as follows: Lay off the scale of the voltmeter tested and the comparison instrument, on the coördinate axes *OA* and *OB*. Plot the curve *OF* between readings of the voltmeter and of the reference instrument, at the frequency for which the voltmeter was designed, using *OA* as the axis of the reference instrument and *OB* as the axis of the test voltmeter readings. From the set of readings at some other frequency, take a reading of the reference instrument, as *OD*. Project this to the curve *OF* at *H*. Draw *HC* parallel to *OD* and produce *CH*. Take the reading of the test voltmeter corresponding to *OD* on *OA*, as *OE*. Project *E* to cut *CH* at *K*. Repeat for other points. *OG* is then a curve between the two sets of readings for the test voltmeter and thus eliminates the reference instrument.

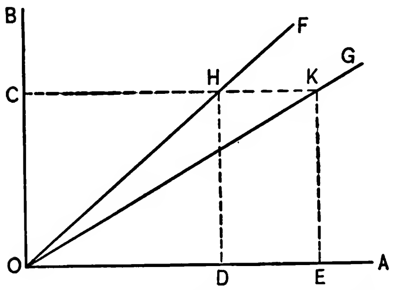

FIG. 20. Calibration curves showing the effect of frequency upon the indications of a voltmeter.

Explain. Carefully, by the use of sketches, the principle upon which the instrument works and the reasons for error in its indications.

Question. What direction of error would result if the temper-

ature of the instrument increased during the test? Is there any device on the instrument for correcting this error?

No. 21. EXPERIMENTAL DETERMINATION OF THE EFFECT OF FREQUENCY ON THE INDICATIONS OF AN AMMETER.

References. *Elect'n Lond.,* November 25, 1904, W. E. Sumpner, The use of iron in alternating current instruments; *Trans. Am. Inst. Elec. Eng.,* June, 1905, E. F. Northrup, A new instrument for the measurement of alternating currents; Gray, pp. 676 to 680; Handbuch der Elektrotech., Vol. 2^4, pp. 224 to 226, Vol. 2^5, pp. 46 and 47.

Theory and Method. Inductance in an ammeter does not cause errors in its indications with different frequencies, as in the case of a voltmeter. If the electro-dynamometer type of instrument is used and heavy non-laminated masses of iron are included in the path of the magnetic flux, the indications of the instrument are affected by variation of frequency. Instruments of other types, such as the magnetic vane, plunger and induction type, are also affected by frequency variation, some to a marked extent. Instruments of the induction type cannot be used on direct current circuits, as their indications depend upon eddy currents induced in some piece of metal, usually an aluminum disc or cylinder. These instruments are generally affected by a change of frequency.

The comparison may be made with a hot wire ammeter or an electro-dynamometer which is free from heavy masses of metal. Instruments of the Kelvin Balance type may also be used.

Data. Compare the ammeter with one whose indications are not affected by change in frequency, using alternating current of two or more frequencies. If possible, calibrate also with direct current, using the same precautions as given in Experiment 20.

Curves. Plot curves as described in Expreiment 20 and de-

termine the error caused by a change of frequency from that for which the instrument was designed.

Explain. By the aid of diagrams, the principle upon which the instrument works and give reasons for errors in its indications. Give the percent. error.

Question. What would be the effect of increase of temperature on the indications of the instrument?

No. 22. EXPERIMENTAL DETERMINATION OF THE EFFECT OF FREQUENCY ON THE INDICATIONS OF A WATTMETER.

References. *Elect'n Lond.*, November 25, 1904, W. E. Sumpner, The use of iron in alternating current instruments; *Trans. Am. Inst. Elec. Eng.*, June, 1905, E. F. Northrup, A new instrument for the measurement of alternating currents; Handbuch der Elektrotech., Vol. 2^4, pp. 224 to 226, Vol. 2^5, pp. 46 and 47; Arnold, Vol. 1, pp. 199 to 202; Gray, p. 680; Bedell, pp. 270 to 375.

Theory and Method. Wattmeters of the electro-dynamometer type have two coils, a fixed coil of heavy wire and a movable coil carrying the pointer and wound with fine wire connected in series with a high non-inductive resistance. The fixed coil carries the line current or some proportional part of this current. The fine wire coil is connected across the line or across a potential transformer which gives it a proportional part of the line pressure. If the inductance of the fine wire coil is inappreciable in comparison with its resistance, the current in it will be in phase with the line pressure. The indications of the instrument will then be proportional to $IE \cos \beta$. In this case the only correction necessary is that for the power consumed in the wattmeter itself. Unless a compensating coil is provided, the reading will include the power lost in either the pressure or the current coil of the meter. In either case the error will be small, and, except

in the case of instruments for reading small amounts of power, may generally be neglected.

If, however, there is appreciable inductance in the pressure coil circuit, the current in the pressure coil is reduced in value and altered in phase by any increase in the frequency of the current. The correction factor will be

$$K = \frac{1 + \omega^2 T_1^2}{1 + \omega^2 T_1 T},$$

in which

$$T = \frac{L}{R} \quad \text{and} \quad T_1 = \frac{L_1}{R_1} \text{ are time constants,}$$

where L and L_1, and R and R_1 are the inductances and the resistances in the measured circuit and the pressure coil, respectively.

This expression reduces to unity when L_1 is negligible or when T_1 equals T. Consequently, the readings will be correct in these two cases. When T_1 is less than T, the correction factor is less than unity, and the wattmeter reads too high, while the reverse is true when T_1 is greater than T. Hence the wattmeter, with appreciable inductance in the pressure coil, may indicate too high or too low, depending upon the value of the inductance in the circuit measured. The only safety lies in the use of a wattmeter with negligible inductance in the pressure coil circuit.

To experimentally determine the magnitude of this error in a given wattmeter, take a non-inductive load, in which case $T = 0$, and the correction factor will be

$$K = 1 + \omega^2 T_1^2.$$

If, under this condition, the true value of power as shown by the product of the voltmeter and ammeter readings, is obtained, the wattmeter may be compared at a given frequency and the correction factor calculated from the equation

$$K = \frac{\text{Power}}{\text{Meter reading}}.$$

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From this value the value of T_1 may also be obtained, by the previous equation. The correction for any other power factor or frequency may then be calculated.

The power taken by the voltmeter may, in general, be neglected or entirely eliminated by disconnecting the voltmeter when the wattmeter is being read. The voltmeter readings should not be affected by the frequency but, if so, the true value may be taken from a calibration curve as obtained in Experiment 20.

Data. Measure the power expended in a non-inductive circuit by means of a voltmeter and ammeter and also by use of a wattmeter. Observe the frequency. Make the same determinations at some other frequency and compare the results. If the wattmeter is affected by frequency, determine the values of T_1 and K for the instrument.

Problem. Apply the correction, as obtained, to a circuit having a time constant of $T = 0.002$.

Curves. Curves should be drawn to show the variation of the wattmeter readings under the conditions of operation. Use true watts as ordinates.

No. 23. STUDY AND CALIBRATION OF AN INTEGRATING AMMETER.

References. Fleming, Vol. 2, pp. 39 to 45, and pp. 73 to 75; Gerhardi, Chap. 2, 4 and 11; Handbuch der Elektrotech., Vol. 2[6], pp. 43 and 65 to 79; Manufacturers' bulletins.

Theory. Integrating meters for the commercial measurement of electrical energy may be divided into two general classes, as follows:

1. Ammeters, strictly ampere-hour meters.
2. Wattmeters, strictly watt-hour meters.

One form of integrating wattmeter has been considered in Direct Current Experiment No. 90. The general discussion given in that experiment will be found to be of value in the per-

formance and discussion of the present experiment as well as those immediately following.

Most integrating ammeters are of the induction type. The torque is produced by a rotary field set up by one or more low reactance coils connected in series with the metered circuit. This field is usually made to act upon either an aluminum cylinder or a disc placed within it and mounted between jewelled bearings. The rotary field is due solely to the current in the metered circuit, being independent of the pressure across the circuit.

Generally, in the production of the rotary field, one or more short circuited secondary coils are employed, these secondaries being inclined or tipped relatively to the primary coil. The torque exerted upon the motor thus becomes proportional to the square of the current in the metered circuit. Some form of retarding device which is positive in its action is necessary, to set up a resisting torque such that the speed of the motor becomes proportional to the current; that is, the resisting torque must vary as the square of the speed. The device generally employed consists of a set of fans or air paddles which are mounted upon the motor shaft. Integrating ammeters may be used in the place of wattmeters on constant pressure circuits, of known voltage, provided the power factor of the circuit is known. They have been used to a considerable extent on incandescent lighting circuits, the power factor being practically unity. One great advantage of the ammeter consists in the fact that it has no pressure coil, which is a source of waste in the wattmeter. However, even the mains supplying incandescent lamps are liable to variation and, generally, it has been found advisable to employ wattmeters in preference to ammeters.

All commercial intergrating meters, whether ammeters or wattmeters, possess an adjusting coil or device for regulating the starting torque and for making the finer adjustments to regulate the speed for the range of load. In ammeters, adjustments may generally be made by changing the angle which the short circuit

coil makes with the plane of the fixed coil. To insure accuracy on light loads, the fans must be carefully balanced.

Meters of the same type, which differ greatly in range, have many parts identical, especially the recording mechanism. It is advisable to operate all meters at about the same speed for rated full load, irrespective of capacity. Hence, a "constant" is stamped upon the dial plate, and the indications of the meter must be multiplied by this constant in order to obtain true readings. Some manufacturers so construct the recording mechanism for all meters that one revolution of the rotor records one ampere hour. Meters of small capacity usually have constants of considerably greater value.

Some meters are made direct reading, by means of changes in the gears of the recording mechanism, thus avoiding the use of a constant. This has been largely because of annoyance occasioned by consumers who do not understand the necessity of the introduction of a constant.

Calibration. In order to calibrate an integrating ammeter it is necessary to test its accuracy for various currents throughout its working range. First ascertain the number of turns of the rotor necessary to record one ampere hour on the dial. The meter is then connected in series with an indicating ammeter and supplied with current of a proper frequency. The current is maintained constant at the desired value and the speed of the rotor is obtained by counting the number of revolutions in a given time. The current is then adjusted to another value and the process is continued. The true constant of the meter for any given current may be obtained by dividing the actual ampere hours, as shown on the dial, by the time of the test.

No. 24. STUDY AND CALIBRATION OF AN INTEGRATING WATTMETER.

References. Standard Handbook, Sec. 3, Art. 283 to 325; Karapetoff, pp. 99 to 104; Franklin and Esty, p. 49; Gerhardi,

Chap. 2, 4 and 11; Fleming, pp. 55 to 88; Handbuch der Elektrotech., Vol. 2^6, pp. 80 to 155; *Elec. Wld.*, June 16, 1906, W. Stanley, A new induction watt-hour meter; Manufacturers' bulletins.

Theory and Method. Probably the simplest type of integrating wattmeter is that illustrated by the Thomson wattmeter, described in Direct Current Experiment No. 90. This meter will run on alternating as well as direct current circuits. The direction of rotation is not changed when both the armature and field currents are reversed. In the Thomson wattmeter, operating on an alternating current with a non-inductive load, the armature and field currents are reversed at the same instant, since both the current and pressure circuits are practically non-inductive. Consequently the torque is at all times exerted in the same direction.

The torque exerted upon the rotor is directly proportional to the power expended in the circuit measured. Since the rotor must speed up to a point where the resisting torque is equal to the impelling torque, it becomes necessary to use some retarding device, the resisting torque of which varies directly with the speed. The retarding device usually employed in wattmeters consists essentially of a copper or aluminum disc (mounted on the shaft of the rotor), which revolves between the poles of a permanent magnet. If the speed is varied, the pressure tending to set up eddy currents in the disc is varied in the same proportion. Since the resistance of the current paths remains unaltered, the resisting torque varies directly with the speed.

In many wattmeters the permanent magnets act upon the rotor itself instead of upon a separate disc. This permits of the rotating part being made lighter, with the result that the meter may be made more compact and the cost of the construction reduced.

The speed of a wattmeter which is used upon an alternating current circuit, should depend upon the product of the current, pressure and power factor. To obtain accurate indications of power on inductive loads, has been one of the great difficulties encountered in the practical construction of meters. In some

of the early meters the construction was such that they would stop, or even run backwards, on inductive loads of low power factor. A serious difficulty in the practical use of wattmeters has been that of the effect of the variation of frequency. At present the general practice is to build meters for a given frequency, and to specify the limit of frequency for a given error.

When an integrating wattmeter of the Thomson type is connected to a non-inductive load, the impelling torque is always in the same direction, as the currents in the current and pressure coils pass through zero at the same instant. On inductive and capacity loads, the impelling torque is reversed twice in a cycle, due to the currents in the current and pressure coils being relatively reversed. The consequence is that the average impelling torque is less than it would be for the same apparent watts when the power factor is unity.

If the pressure and current coils may be considered as non-inductive, the average impelling torque will be directly proportional to the pressure, current and cosine of the angle of phase difference, whether operating under conditions of non-inductive, inductive or capacity load.

If the pressure coil has appreciable self-inductance, and the self-inductance of the current coil is negligible, the meter will tend to run too fast on inductive loads and too slow on capacity loads, assuming it has been adjusted for accuracy on non-inductive loads. This would be due to the currents in the two coils becoming more or less nearly in phase, the maximum rotary effect for a given current occurring when they are exactly in phase. If the reactance becomes so great that the current in the current coil lags behind that in the pressure coil, the rotary effect diminishes, although the meter will still run too fast since the phase difference between the impressed pressure and the current of the metered circuit is greater than that between the currents in the pressure and current coils of the meter. The self-inductance of the current coil is very small and may, in general, be neglected, although its effect would be to slightly increase the error introduced.

Wattmeters may be tested on inductive and capacity loads, by connecting them directly into circuits which contain inductances or capacities of known value and making the test in the general manner used for non-inductive circuits. A method which is much more flexible is as follows. A double alternator is used, consisting of two machines mounted on the same bed plate and arranged so that the armatures or fields may be shifted mechanically at any desired angle. One of these armatures is connected to the pressure coil of the wattmeter, while the other supplies the current coil. This armature may be short circuited on the current coils of the instruments or, preferably, connected through these instruments to a non-inductive load. By varying the angular position of the armature coils of the two machines, relatively, any desired phase angle may be obtained between the currents in the pressure and current coils of the wattmeter. This method may also be used for meters of high range as follows. The armature of one of the machines is connected to the low voltage coil of a transformer (if it is desired to step up the voltage) and the high voltage coil of the transformer is connected across the pressure coil of the wattmeter or its transformer. The low current coil of a current of welding transformer is short circuited across the other armature, and the high current coil connected to the current coil of the wattmeter. By regulating the fields of the two alternators, the calibration may be made with the loss of very little power.

The value of this phase angle may be approximately determined by the angular displacement of the two armatures, but the power factor should be obtained by taking voltmeter, ammeter and wattmeter readings and substituting in the formula

$$\text{Power factor} = \frac{\text{watts}}{\text{volts} \times \text{amperes}}.$$

All commercial integrating meters possess some form of adjusting coil or device for regulating the starting torque and for making the finer adjustments for calibration over a range of fre-

quency and power factor, so that the meter, after it is set up, may be carefully tested and adjusted. If you are testing a meter with the object of purchasing or rejecting the same, the functions of such devices on the meters tested should be studied and understood as far as possible.

Meters of the same type, which differ greatly in range, have many parts identical, especially the recording mechanism. It is advisable to operate all meters at about the same speed for rated full load, irrespective of their capacities. Hence, a "constant" is stamped upon the dial plate and the indications of the meter must be multiplied by this constant in order to obtain the true reading. Some manufacturers so construct the recording mechanism for all meters that one revolution of the rotor records one watt hour. Meters of small capacity usually have constants of greater value.

Some meters are made direct reading by means of changes in the gears of the recording mechanism, thus avoiding the use of a constant. This has been done largely because of annoyance occasioned by consumers who do not understand the necessity of the introduction of a constant.

Calibration. Before starting the calibration, ascertain the number of turns of the rotor necessary to record one watt hour on the dial. This is usually unity.

In calibrating the meter, it should be tested throughout its entire working range. When calibrating on a non-inductive load it is not necessary to use an indicating wattmeter, an ammeter and voltmeter being sufficient. The meter should be connected to a current source of the proper frequency and pressure, and these two factors should be kept constant throughout the test. The current should be maintained constant at the desired value and the speed of the motor obtained by finding the time necessary to make a given number of revolutions. The current is then adjusted to another value and the process continued.

The same test should be made on inductive and capacity loads of different power factors if the meter is intended to be used

under such conditions. In making these tests an indicating wattmeter is necessary.

It is desirable to test the meter upon frequencies above and below normal; also for small variations in pressure. The true constant of the meter for any given load may be found by dividing the actual watt hours by the watt hours as shown on the dial for the time of the test.

Data. Maintaining the frequency and pressure constant at their normal values, calibrate the instrument by taking a series of observations at currents ranging from about 50 percent. above normal full load down to the least current which will keep the rotor moving. Also determine the least current which will start the rotor.

If possible, repeat the tests with inductive and capacity loads of 80 percent. power factor. Also find the effects of variation of frequency and of pressure, on the accuracy of the meter.

Compute. The true constant of the meter for each load taken; also compute the percent. error introduced by using the constant given by the maker instead of the true constant of the meter.

Curves. Using percents of normal full load current as abscissas, plot curves of true constant and of percent. error.

Explain. The working principle and the adjustments of the meter.

Questions. How does a rise in temperature affect the accuracy of the meter? How does a change in frequency affect the accuracy of the meter? In the answers to these questions give the reasons as far as possible. How does a change in voltage affect the accuracy of the meter? What type of a wattmeter would run backward on an inductive load of low power factor? When would an induction meter run fastest upon a capacity load, the current and pressure remaining constant?

What factors might cause a meter to be inaccurate on rapidly fluctuating loads, even though it records accurately on loads of constant value throughout the range of fluctuation? Would you use a high resistance in series with the pressure coil of a 110-volt

induction meter for use on a 440-volt power and lighting circuit? Could the meter tested be used safely on a lighting circuit of the same voltage but of different frequency?

No. 25. TEST OF AN INTEGRATING WATTMETER WHEN USED WITH TRANSFORMERS.

References. Fleming, Vol. 2, p. 89; Gerhardi, Chap. 2, 4 and 11; Manufacturers' bulletins.

Theory and Method. When integrating wattmeters are used on alternating circuits of 500 volts or less, the pressure circuit is generally connected directly across the mains. Again, if the current metered on such circuits is not in excess of about 100 amperes, the practice is to connect the current coil of the instrument directly in series with the metered circuit. Should the pressure be 500 volts or less, and the current over 100 amperes, it is customary to make use of a series or current transformer, the primary of which is placed in series with the circuit to be metered, while the secondary is short circuited upon the current coils of the meter. The ratio of current transformation depends upon the maximum current in the metered circuit, as it is customary to construct these transformers for a certain maximum secondary current, thus permitting of the same wattmeter being adaptable to various conditions.

It frequently occurs that it is desired to meter circuits which have pressures greater than 500 volts. In such cases, provided the pressure is not more than 2,000 volts and the maximum current is not large, some companies connect the current coils directly in series with the mains, and connect the pressure coils across the secondary of a pressure transformer, the primary of which is connected across the metered circuit. The pressure transformers are ordinarily wound for 100 volt secondaries, the ratio of transformation depending upon the pressure of the metered circuit. When the latter is 2,000 volts, or above, an oil insulated transformer is generally used.

Many of the manufacturers prefer to use both current and pressure transformers for all cases where the pressure is higher than about 500 volts. This arrangement has the decided advantage of insulating the instrument from the high pressure mains. In all cases where meters are used with transformers, they should be calibrated in connection with these transformers. This is essential because the ratio of transformation of different transformers, even of the same size and type, varies somewhat and because the phase difference of primaries and secondaries may not be exactly 180 degrees.

In calibrating, the same general method of procedure is employed as was indicated in Experiment 24. The standard indicating wattmeter is connected directly into the metered circuit, to which the meter under test is connected through its proper transformers. The usual precautions should be taken to guard against static effects.

Data. Test the wattmeter for various loads from one-half overload down to the least load at which the rotor will revolve. Tests should also be made of the least load at which the rotor will start. The frequency and pressure should be maintained constant at their normal values. If the instrument is to be used on an inductive or a capacity load, it should be tested under such conditions.

Compute. The true constant of the meter for each load taken; also the percent. error introduced by using the constant given by the manufacturer instead of the true constant.

Curves. Using percent. of normal full load current as abscissas, plot curves of true constant and of percent. error.

Explain. The fundamental principles and assumptions made in the application of current and pressure transformers to meter circuits.

In what way errors might be introduced if several instruments were used in connection with the same transformer.

Question. A meter has been calibrated for use independent of transformers and is used in connection with transformers of

known ratios. Will its readings be more nearly accurate when used with both pressure and current transformers or when used with either alone? Why?

No. 26. STUDY OF THE CONSTRUCTION OF A TRANSFORMER.

References. Bedell, Chap. 15; Karapetoff, Chap. 19; Esty, pp. 239 to 260; Franklin and Esty, Chap. 10; Handbuch der Elektrotech., Vol. 5, pp. 726 to 736; Arnold, Vol. 2, Chap. 12; Thompson's "Dynamos," Vol. 2, Chap. 12 and 13.

Object. To determine the mechanical construction and the magnetic circuit and winding data of a commercial transformer.

Theory and Method. It sometimes becomes necessary to check the design data and construction of a transformer after it has been assembled. The method followed in this experiment can be applied, with the necessary modifications, to almost any class of electrical machine.

The general features of construction of a transformer tending to its proper operation in one class of service or another may be determined by a superficial examination. These features include the protection of the electrical circuits against mechanical injury and the exclusion of moisture from the coils. Other necessary features to determine are the weight and volume of the apparatus for the rated output, and the ease with which it may be installed for service.

The magnetic circuit may be determined from actual measurements of the core as assembled. From these figures the probable losses of the device may be calculated by the application of the constants for the iron in use at the time the transformer was constructed. The method of assembling and insulating the laminations from each other to prevent eddy currents, is also important.

The resistance of each coil of the primary and of the secondary should be determined, in order to ascertain the possibility of oper-

ating the coils in multiple and to check in a rough manner the number of turns on each coil. The number of turns on each coil may be determined if the turns on one coil are known, by applying a known alternating pressure to this coil and reading the pressure generated in the other coils. A search coil of a known number of turns may be wound on the core and the ratio determined for each coil with this coil. If the coils are already wound and in place, it is not always possible to determine the size and insulation of the conductors without injuring the coils. The effective area of the copper in each coil may be determined indirectly from the resistance of the coil at a known temperature, the number of turns and the approximate length of a mean turn as determined by mechanical dimensions. If this does not correspond to any size as given by a wire table, a combination of two or more wires of some smaller size in multiple may check with the area as found.

With the winding data and the dimensions of the magnetic circuit known, the other constants of the transformer may be calculated. These will include the magnetic induction in the core and the losses in the core, using the proper values of constants to fit the quality of iron used at the time the transformer was manufactured. An alternating pressure of the proper value and frequency should be applied to one of the windings of the transformer and the exciting current and core loss measured. These values may be used to check the calculated values.

Data should be taken as indicated above and as outlined on the schedule given below. The report should include the results tabulated as indicated; also sample calculations to show how they are derived.

SCHEDULE OF TRANSFORMER DATA.

I. Rating of the transformer.

Type ..A...., No. [illegible], Specification No. ....,
Kw. ..1....., Volts, pri. ......, sec. ......, Freq. ......

II. Features of construction.

Form of containing case ....
Method of excluding moisture ....

Method of fastening transformer in case ....
Method of installing for use ....
Method of cooling ....
Method of bringing out leads, pri. ......, sec. ....
Method of ventilation, core ...., coils ....
Method of prevention of oil leakage at leads ....
Are there fuse blocks attached? ....
Method of protection when installed ....
Other features ....
Sketch of connection board and method of connection.

III. Magnetic circuit.

Sketch showing dimensions on page — .... of report
Type of core ....
Thickness of laminations ....
Sketch and dimensions of each stamping on page — of report
Sketch showing method of assembling on page — of report
Insulation between core plates ....
Number of magnetic joints ....
Number of magnetic circuits ....
Gross volume of core, cu. cms..........................
Net volume of core, cu. cms............................
Net weight of core (.017 lbs. per cu. cms.), lbs............
Net weight per kw. capacity, lbs......................

IV. Winding data.

Number of turns in coil ....pri........., sec.......
Number of turns, total ....pri........., sec.......
Average length of turn ....pri........., sec.......
Total length of wire ....pri........., sec.......
Total weight of wire, lbs.......pri........., sec.......
Resistance, ohms at 25° C. ......pri........., sec.......
Size of conductors in cir. mils ...pri........., sec.......
Full load current, amperes ......pri........., sec.......
Current density, cir. mils per amp..pri........., sec.......
Current density, amps per sq. in. .pri........., sec.......

Insulation on wire . . . . .pri. . . . . , sec. . . . . . .
Insulation on coil . . . . .pri. . . . . , sec. . . . . . .
Insulation between coils. . . . .
Winding space . . . .
Space between coils and case . . . .
Space between core and case . . . .

V. Calculation of magnetic induction, based on . . . . winding.
No. turns . . . . , emf. . . . . , cycles . . . .
Total max. flux = effective volts $\times$ 100,000,000 $\div$ 4.44 $\times$ cycles $\times$ turns . . . .
Gross cross section of magnetic circuits sq. cm. . . . .
Net cross section (. . . . . . percent. of gross) sq. cm. . . . .
Maximum flux per sq. cm. (density $\mathcal{B}_{max}$) lines . . . .
Estimated loss in cu. cm. per cycle per sec., watts . . . .
Estimated loss in . . . . . . cu. cms. at . . . . cycles, watts . . . . .
Energy current at . . . . . . volts for iron loss, amperes. . . . . .
Observed loss by wattmeter, watts . . . .
Total no load current, amperes . . . .
Effective magnetizing current, amperes . . . .
Maximum magnetizing current, amperes . . . .
Maximum magnetizing ampere turns. . . . .
Length of magnetic circuit, centimeters . . . .
Ampere turns per cm. length for $\mathcal{B}_{max}$ . . . .

No. 27. METHODS OF CONNECTING THE COILS OF SINGLE TRANSFORMERS.

References. Bedell, pp. 15 and 16; Franklin and Esty, pp. 217 and 218; Karapetoff, Chap. 19; Esty, pp. 239 to 260; Handbuch der Elektrotech., Vol. 5, pp. 726 to 736; Arnold, Vol. 2, Chap. 12; Thompson's "Dynamos," Vol. 2, Chap. 12 and 13.

Object. To study the application of the principles of transformer windings.

Theory and Method. Most commercial transformers for

electric lighting are supplied with two primary and two secondary windings as represented in Figure 27*A*, where the primaries are each wound for 1,100 volts and the secondaries for 110 volts.

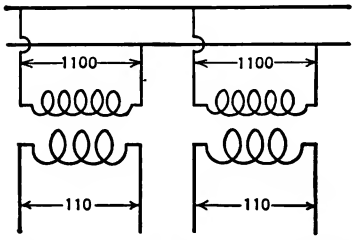

FIG. 27*A*. Transformer, primaries in multiple.

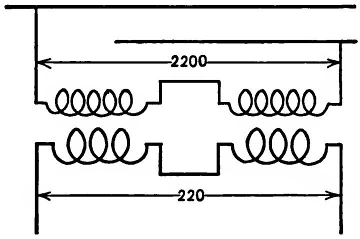

FIG. 27*B*. Transformer, primaries in series, secondaries in series.

The primaries may be connected in series as shown in Figure 27*B*, and the transformer is then suitable for 2,200 volt mains. The same figure shows the correct connection for the delivery of current at 220 volts. Should the primaries be connected as shown in Figure 27*C*, they would tend to magnetize the core in opposite directions and the resultant magnetization would be practically nil. The coils would then have but little counter pressure, and would burn out unless properly fused. The danger is not great in the case of a core type transformer in an iron case, on account of the excessive magnetic leakage under this condition. The secondaries are shown as developing zero pressure. This is not strictly true because the opposed primaries would establish consequent poles and some lines of force would pass through the leakage paths and the coils, but so far as prac-

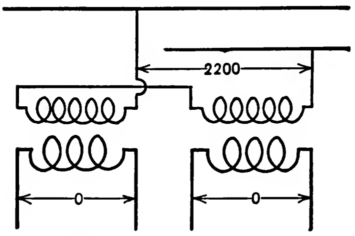

FIG. 27*C*. Transformer, wrong connection, primaries in series but in opposition.

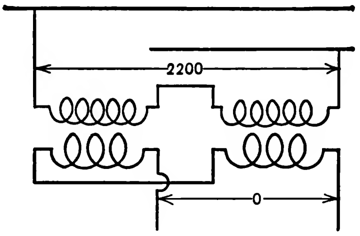

FIG. 27*D*. Transformer, wrong connection, secondaries in opposition.

tical consequences are concerned this pressure might as well be zero. In Figure *27D*, the primaries are properly connected for 2,200 volt service, but the secondaries are opposed. The result is zero secondary pressure, and if any attempt is made to parallel this secondary with that of a properly connected transformer, the secondaries of both transformers and the primary of the correctly connected one will burn out if the primaries are not properly protected by fuses.

Figure *27E* indicates the correct method of connecting the primary windings in parallel for 1,100-volt service. In Figure

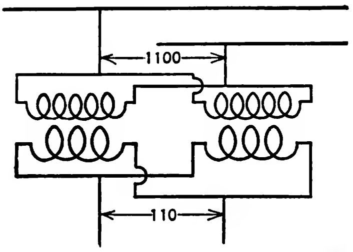

Fig. 27*E*. Transformer, primaries in multiple, secondaries in multiple.

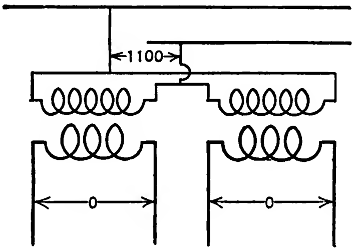

Fig. 27*F*. Transformer, wrong connection, primaries in multiple but in opposition.

27F, the primaries are in parallel, but in opposition. Unless the primaries are properly fused, they will both burn out, as in Figure *27C*.

In Figure *27G* the primaries are properly connected in parallel, but the secondaries are not. The secondary connections in this case are practically the same as that of Figure *27B* short circuited.

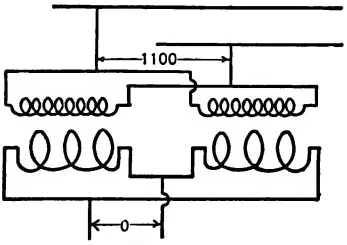

Fig. 27*G*. Transformer, wrong connection, secondaries in opposition.

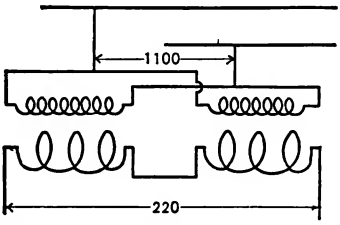

Fig. 27*H*. Transformer, primaries in multiple, secondaries in series.

Figure 27*H* shows the correct connection of the secondaries for 220 volts. Figure 27*I* indicates a method of connection of secondaries for three-wire service. The primaries may be connected either as shown or as in the preceding diagram; however, this point is more specifically treated in another experiment.

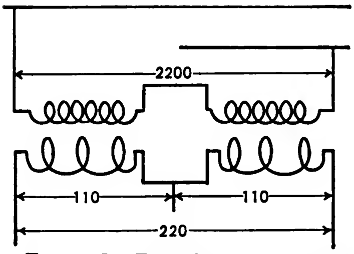

FIG. 27*I*. Transformer, primaries in series, secondaries connected for the three wire system.

From what has been said it is evident that troublesome mistakes can be easily made, and to avoid these the manufacturers send out a card with each transformer, explaining the connections. Some mark the transformer terminals.

Data. Select a transformer with two primary and two secondary windings. Fuse the primary, using fuses of a capacity not greater than the full load primary current when the two primary coils are connected in series. Connect the primaries in series and to a 110-volt alternating current circuit through a 110-volt incandescent lamp. If the two primary coils are opposed, the lamp will glow at its normal candle power, while if the transformer is connected correctly the lamp will glow but little or not at all. The normal current of the lamp should be less than the leakage current of the transformer at 110 volts. The success of this method depends, somewhat, upon the form of the magnetic circuit and upon the relative position of the primary coils. If the connection has not been properly made the first time, reverse the connections of one of the primary windings and repeat the test. Mark the terminals of each of the primary leads for identification. Make connections as in Figure 27*B* and impress 2,200 volts on the primary. By means of lamps or a voltmeter, find out if the secondary pressures are in conjunction or in opposition. Mark the secondary terminals. Make the following additional tests:—

1. Connect according to Figure 27*I*,

2. Connect according to Figure 27*D*,
3. Connect according to Figure 27*C*,
4. Connect according to Figure 27*H*,
5. Connect according to Figure 27*E*,
6. Connect according to Figure 27*G*,
7. Connect according to Figure 27*F*.

Devise. A test for identifying transformer terminals, using fuses only; another, using a primary cell and a direct current voltmeter as a ballistic galvanometer.

Show. In which circuit the fuses of a transformer should be placed in order to protect it the more effectually and why.

No. 28. METHODS OF CONNECTING TWO OR MORE TRANSFORMERS FOR COMBINED OUTPUT.

Reference. Bedell, pp. 11 to 14.

Object. In this experiment an investigation is made of the various ways in which two or more transformers may be connected for combined output.

Theory and Method. In Experiment 27, not only was it necessary to combine two secondary pressures in such a manner as to avoid short circuits and still get the full capacity of the transformer, but it was also imperative that the magnetic pressures due to the primary windings be made additive. In the present case each transformer is assumed to be correctly connected, and its primary and secondary windings are each dealt with as a unit. In Figure 28*A*, two transformers are connected to a distribution system, with their primaries in parallel and their secondaries in parallel. This is the most common method used in practice. The two primaries may be connected to the line without reference to polarity and a polarity test with lamps or a voltmeter should then be applied to the secondaries. The secondary pressures should be opposed. Another method is to

use light primary fuses and to connect up the secondaries without regard to polarity. If the fuses blow, the secondary termi-

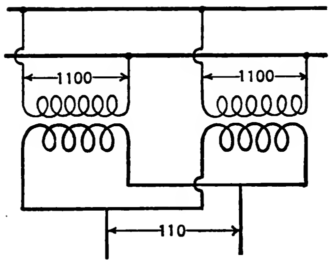

FIG. 28*A*. Two transformers, primaries in multiple, secondaries in multiple.

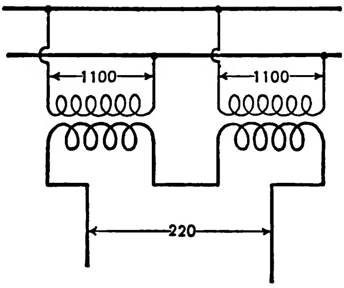

FIG. 28*B*. Two transformers, primaries in multiple, secondaries in series.

nals of one of the transformers should be reversed. Reversing the primary of one transformer would produce the same effect, but it is never advisable to cross the high voltage terminals and transformers are all designed so that this may be avoided.

In Figure 28*B*, the secondaries are additive. In practice this method is also common. Doubling the pressure of a distribution system quadruples the distance which can be reached from one distribution center, the line loss and amount of copper remaining the same.

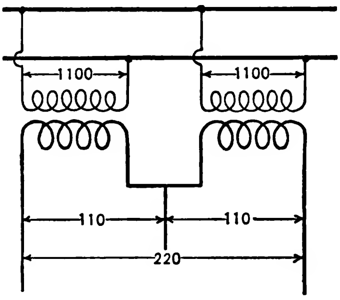

FIG. 28*C*. Two transformers, primaries in multiple, secondaries connected for the three wire system.

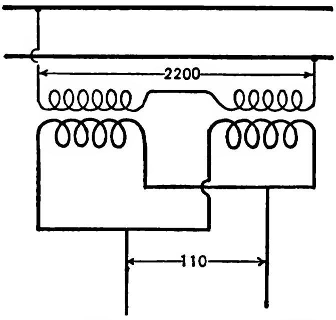

FIG. 28*D*. Two transformers, primaries in series, secondaries in multiple.

Figure 28*C* is a modification of Figure 28*B*, in that a three-wire secondary system is used, the neutral being connected at

the junction point of the two secondaries. When motors are operated on this system they are usually connected across the outside wires.

Figure 28*D* shows a method of connection for combined output, the primaries being in series. Some stations have a number of old style 1100 to 110-volt transformers which have a fair efficiency. Should it become necessary to change to a 2200-volt system, this method might be adopted and the old transformers utilized. It is obvious that a number of methods of connecting secondaries could be used.

Data. Connect two transformers as in Figure 28*A,* using suitable fuses. Make the polarity tests mentioned above. Reverse the connections of one of the primaries and note the result. Reverse one of the secondaries, leaving the primaries as last connected. Connect, successively, as in Figures 28*B,* 28*C* and 28*D,* and go through the same operation, noting carefully the division of load in each case.

Suggestion. If two transformers of the same capacity are available, it will be interesting to load each operative combination and to determine the division of the load with instruments.

Explain. If it is possible to operate two transformers of different capacity each at full load, the connections being made successively according to Figures 28*A,* 28*B,* 28*C* and 28*D.*

No. 29. VARIATION OF THE REACTANCE OF A COIL CONTAINING IRON IN ITS MAGNETIC CIRCUIT FOR VARIOUS VALUES OF CURRENT.

References. Bedell, Chap. 13; *Jour. Worcester Poly. Inst.,* January, 1901, H. B. Smith; *Zeitschr. f. Elektroteck.,* August 14, 1904, F. Spielman, The computation of alternating currents without assuming a constant coefficient of self-induction.

Theory and Method. When a coil contains iron in its magnetic circuit, the value of its inductance is no longer constant

but depends upon the permeability of the iron. The permeability of iron being greater than that of air, the inductance of the coil is increased by the presence of iron. If the iron core is continuous, the following relation will be found to hold:—

$$L' = \mu L, \tag{29a}$$

where

L = the inductance without iron or core,
L' = the inductance with iron or core, and
μ = the permeability of the iron.

Since the permeability of iron is a variable quantity depending upon the degree of magnetization, and hence upon the current in the coil, the value of the inductance also depends upon the current.

Referring to Figure 29, the induced electromotive force E_1'

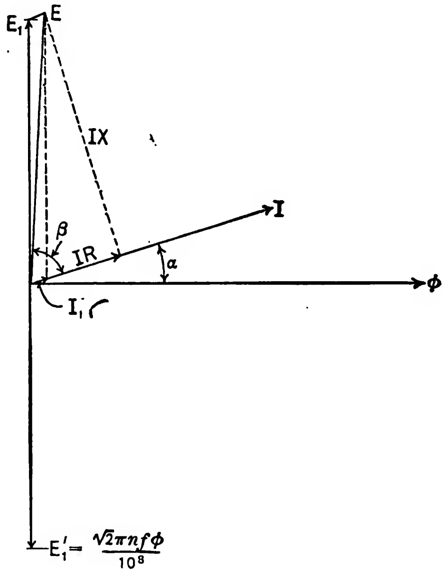

FIG. 29. Diagram showing the relation between current, emf. and flux in a coil containing iron.

will be at right angles with the flux ϕ and of a value such that

$$E_1' = \frac{\sqrt{2}\pi f n \phi}{10^8}, \tag{29b}$$

where n is the number of turns in the coil. A pressure E_1 will be required to overcome the electromotive force E_1'. To produce the flux ϕ the current I will be required, determined by the reluctance of the magnetic circuit and the number of turns in the coil. Due to hysteresis and eddy currents in the iron core, this current will be in advance of the flux by the angle α. There will also be a drop in the coil in phase with I, due to the resistance of the coil. The vector sum of E_1 and Ir gives E, the total electromotive force required to send the current I through the coil.

If components of E are taken in phase with and at right angles to I, there will result IR and IX. R and X, determined from these components, are known as the equivalent resistance and equivalent reactance of the coil. The value of R obtained in this manner, is the same as that obtained from the wattmeter reading, or

$$R = \frac{P}{I^2}.$$

Hence, to find the equivalent resistance and reactance of the coil, a wattmeter should be included in the circuit and the value of R determined. Then, since

$$Z = \frac{E}{I},$$

and

$$X = \sqrt{Z^2 - R^2},$$

X, the equivalent reactance, may be determined in this manner.

The true value of the inductance must be determined from the induced electromotive force E_1. Or, since

$$L' = \frac{n\phi}{10^8 I},$$

it follows that

$$L' = \frac{E_1}{2\pi f I}, \qquad (29c)$$

from equation (29b). E_1 is the vector difference between E and

the resistance drop Ir. Or

$$E = \sqrt{(E_1 + Ir \sin \alpha)^2 + (Ir \cos \alpha)^2},$$

$$E_1 = [\sqrt{E^2 - (Ir \cos \alpha)^2}] - Ir \sin \alpha. \qquad (29d)$$

In most cases the terminal pressure may be used in the place of E_1 without appreciable error. The value of L' must then vary directly with the permeability of the iron as shown by equation (29a). The equivalent resistance and reactance will also vary with the value of the exciting current.

In the ordinary transformer without load, the value of the Ir drop is negligible in comparison with the impressed pressure. Hence

$$\beta + \alpha = 90^\circ \quad \text{(nearly)}.$$

From this the following relations are derived,

$$IX = E_1 \sin \beta,$$

$$I_{\text{mag.}} = I \cos \alpha,$$

$$= I \sin \beta.$$

Then, since

$$2\pi f L = \frac{E_1}{I},$$

$$= \frac{E_1 \sin \beta}{I \sin \beta} = \frac{E_1 \sin \beta}{I \cos \alpha},$$

it will be seen that

$$\frac{X}{2\pi f L} = \frac{I_{\text{mag.}}}{I} = \cos \alpha = \sin \beta,$$

or,

$$X = 2\pi f L \sin \beta,$$

and

$$2\pi f L = \frac{X}{\sin \beta}.$$

Hence the true reactance bears to the equivalent reactance a ratio equal to one divided by the sine of the angle of lag of the current

behind the pressure. This also indicates another method of computing the true inductance of a coil.

If an air gap is introduced into the magnetic circuit, the value of the current will be increased for a given impressed pressure, since the reluctance of the magnetic circuit has been increased. Hence the value of the angle α will be decreased, and X becomes more nearly equal to $2\pi fL'$ for the coil. If the iron is worked at low values of density, the coil will behave very much like an air cored coil.

Data. Take a coil, as the secondary of a transformer, and vary the impressed pressure at normal frequency, from a low value to one about double normal voltage. Take readings of voltage, current and watts absorbed in the circuit. Measure the resistance of the coil. If possible, introduce an air gap in the magnetic circuit and repeat the readings. This may be done by using a coil whose magnetic circuit is specially constructed with a portion of the iron made removable.

Calculate. The values R, X, L and Z for the various values of current .

Curves. Using R, X, L and Z as ordinates, plot curves with current as abscissas.

Question. Why does the curve between L and I take the form found? At what value of current does the iron reach the saturation point? If the coil is that of a transformer, determine from the curves whether the iron is being worked at the proper density.

Caution. If a transformer is used, work with the low tension winding. Carefully tape up the high tension winding to protect it from accidental contact with the body, as there may be dangerous voltages there at times.

No. 30. VARIATION OF THE INDUCTANCE, REACTANCE AND IMPEDANCE OF A TRANSFORMER ON OPEN CIRCUIT.

(*a*) With Pressure, Magnetization Remaining Constant,
(*b*) With Frequency, Magnetization Remaining Constant,
(*c*) With Frequency, Pressure Remaining Constant,
(*d*) With Pressure, Frequency Remaining Constant.

References. Bedell, pp. 44 and 45 and 84 to 86; Thompson's "Dynamos," Vol. 2, pp. 535 to 536; Steinmetz' "A.C. Phenomena," p. 235; Arnold, Vol. 1, pp. 219 to 221; *Elec. Rev. Lond.*, July 8, 1905, M. A. Sammett, Operation of a transformer at varying frequencies and voltages.

Object. To study the effect of variation of some of the factors entering into the design and operation of a transformer.

Theory and Method. In Experiment 29 some methods of measuring the self inductance, reactance and impedance of a coil containing iron in its magnetic circuit, were considered. Reference should be made to Experiment 29 for methods of measurement and calculation.

From the equation

$$E_1 = \frac{2\pi f n' \phi}{10^8}, \tag{30a}$$

it is seen that

$$\phi = k\frac{E_1}{f}, \tag{30b}$$

where k is a constant. In Figure 29 it may be seen that E_1 differs from E by a small amount.

From Equation (30*b*) it may be seen that the magnetization will remain constant, providing that E and f are both varied in the same ratio. When armature reactions are small, which will be the case if the only load on the alternator is a transformer whose exciting current is small in comparison with the full load current of the alternator, this may be accomplished by keeping the field excitation constant and varying the speed of the alternator.

From Equation (30*b*) it will also be seen that if E_1 is kept constant, ϕ will vary inversely with the frequency. Also, if f is maintained constant, the magnetization will vary directly with E_1. A convenient method of measuring E_1 on large transformers is by the use of a pressure transformer and a voltmeter across the high tension side, the other reading being taken from the low tension side.

Data. Measure the self inductance, reactance and impedance of a transformer from the low tension side, by the method given in Experiment 29, and under the following conditions of variation.

(1) With constant magnetization, pressure and frequency varying in the same ratio.

(2) With constant pressure, frequency varying.

(3) With constant frequency, pressure varying.

Curves. Plot curves with L, X and Z as ordinates showing conditions (*a*), (*b*), (*c*) and (*d*) of the experiment.

Explain. Why the curves for L take the particular form found.

No. 31. VARIATION OF THE CORE LOSS OF A TRANSFORMER.

(*a*) With Pressure, Magnetization Remaining Constant,
(*b*) With Frequency, Magnetization Remaining Constant,
(*c*) With Frequency, Pressure Remaining Constant,
(*d*) With Pressure, Frequency Remaining Constant.

References. Bedell, pp. 311 to 314; Flemming, Vol. 2, pp. 585 to 591; Sever and Townsend, p. 148; Thompson's "Dynamos," Vol. 2, pp. 578 to 591; Steinmetz' "A.C. Phenomena," pp. 180 to 199; Russell, Vol. 1, pp. 34 to 39, Vol. 2, p. 254; Fleming's "Transformers," Vol. 2, pp. 479 to 500; Arnold, Vol. 2, Chap. 5 and 15; Bull. Univ. Wis., Vol. 1, No. 11, A. H. Ford; *Elec. Rev. Lond.*, July 8, 1905; M. A. Sammett, Operation of a transformer at varying frequencies of voltages.

Object. To investigate some of the factors affecting the core loss in a transformer.

Theory and Method. The core loss of a transformer consists of two factors, the hysteresis loss and the eddy current loss. Let

P_h = the hysteresis loss in watts,
P_f = the eddy current loss in watts,
$P = P_h + P_f$ = the total core loss,
f = the frequency,
$\mathcal{B}_m$ = maximum flux density, and
k_h and k_f = constants.

Then

$$P_h = k_h f \mathcal{B}_m^{1.6}, \tag{31a}$$

and

$$P_f = k_f f^2 \mathcal{B}_m^2, \tag{31b}$$

where

$$\mathcal{B}_m = k \frac{E_1}{f}, \tag{31c}$$

from Equation (30*b*). In commercial transformers P_f is usually small in comparison with P_h.

An inspection of the equation will show that, with constant flux density, the core loss varies according to some power of the frequency between 1 and 2. By substitution of the value of $\mathcal{B}_m$ from Equation (31*c*), it will be seen that, with constant frequency, the core loss varies as some power of the pressure between 1.6 and 2. With constant pressure the eddy current loss is constant and the hysteresis loss varies as the −0.6 power of the frequency, hence decreases with an increase of frequency.

Data. If Experiment 30 has been performed, use the data for the present experiment. Make as wide a variation of the quantities involved as is feasible, considering the safety of the apparatus and the facilities available. The method of holding the flux density constant is given in Experiment 30.

Curves. Plot curves with core loss as ordinates for conditions (*a*), (*b*), (*c*) and (*d*) of the experiment.

Questions. Does each of the curves have the shape expected from an investigation of the equations? Do the eddy current losses seem of much importance?

No. 32. EFFICIENCY OF A TRANSFORMER ON A NON-INDUCTIVE LOAD BY MEASUREMENT OF OUTPUT AND INTAKE.

References. Bedell, pp. 369 to 374; Fleming, Vol. 2, pp. 585 to 591; Sever and Townsend, p. 152; *Elec. Wld. and Eng.*, May 17, 1902, R. F. Schuckart, Transformer testing by central station companies; *Elec. Rev. Lond.*, September 17, 1904; S. E. Johannsson, Transformer testing for central stations; *Elec. Wld. and Eng.*, April 8, 1905, C. W. Humphrey, Transformer practice; *Elec. Jour.*, August, 1905, Will Nesbit, Central station transformer testing; *Elec. Eng. Lond.*, February 7, 1907, E. A. Reid, Practical notes on transformer testing; *Trans. Am. Inst. Elec. Eng.*, June, 1907, H. W. Tobey, Notes on transformer testing.

Object. Efficiency is defined as the ratio of the output to the intake. This experiment measures these quantities directly.

Theory and Method. The output and intake of a transformer may be measured readily, and with a fair degree of accuracy. As there are no moving parts in a transformer, these quantities may be measured with electrical instruments. The intake should be measured by means of a wattmeter. The voltage across the primary should be kept constant throughout the experiment.

Connections should be made as in Figure 32, a potential transformer being used with the voltmeter and wattmeter in the primary circuit, or, if the voltage is low enough, multipliers may be used with these instruments. In the diagram, the power taken by the potential transformer is measured by the wattmeter. This is of constant value, and may be obtained by disconnecting the transformer primary and reading the wattmeter when supplying the potential transformer alone. If the potential transformer had been placed on the other side of the current coil of

the wattmeter a correction would have to be made for the loss in the current coil of the meter.

Transformers should be tested to at least full load, and preferably to about 50 percent. overload. This experiment has

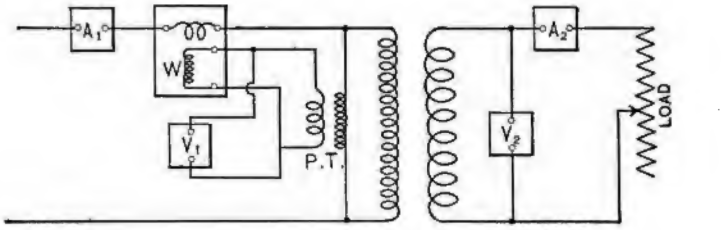

FIG. 32. Connections for determining the efficiency of a transformer by loading.

some disadvantages as to accuracy because of the magnitude of the quantities to be measured and the direct effect of errors on the final result.

Data. Take a set of readings from no load to 50 percent. overload, the primary pressure being maintained constant at its normal value throughout the test.

Curves. Plot curves of efficiency and power factor, taking percent. load as abscissas.

No. 33. EFFICIENCY OF A TRANSFORMER BY THE STRAY POWER METHOD.

References. Bedell, p. 381; Fleming, Vol. 2, pp. 585 to 595; Karapetoff, p. 434; Russell, Vol. 2, pp. 254 to 262; Lamb, pp. 68, 71, and 86 to 90; Standard Handbook, Sec. 6, Art. 36 to 38, and 93 to 100; Handbuch der Elektrotech., Vol. 2^4, p. 218; Arnold, Vol. 2, p. 284; *Bul. Univ. Wis.*, Vol. 1, No. 11, A. H. Ford; see also references to Experiment 32.

Object. To measure the losses in a transformer and to calculate the efficiency from these losses.

Theory and Method. This method of determining the efficiency of a transformer is the one most used in practice be-

cause of its simplicity and accuracy and also because it requires but a small amount of power to make the test. It consists essentially of two tests, the determination of the iron losses and the determination of the copper losses.

The iron losses, consisting of hysteresis and eddy current losses in the laminations, are determined under normal conditions as to voltage and frequency. They are usually determined by applying normal voltage to the low voltage winding of the transformer, and measuring the power intake at normal frequency by means of a wattmeter. It is better to determine these losses by varying the voltage over a range from below normal to above normal by a sufficient amount to be able to plot a curve from the readings showing the relation between voltage and losses. The loss at normal voltage is then taken from this curve. Connections for this test are shown in Figure 33*A*. In making this test the voltage should be applied directly to the transformer without the use of a series resistance. If this is impossible, a resistance absorbing an amount of power greater than that used by the transformer, may be shunted across between the series resistance and the instruments and the pressure across the combination may be controlled by the series resistance. By this means, distortion in the pressure wave and in the flux wave, is avoided.

Since the power measured is that absorbed by the iron of the transformer and depends upon the magnetic induction and fre-

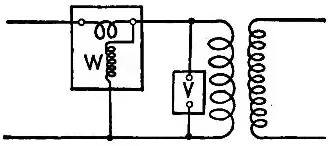

Fig. 33*A*. Connections for measuring the core loss of a transformer.

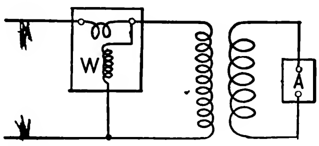

Fig. 33*B*. Connections for measuring the copper loss of a transformer.

quency, the same losses would be measured from either side of the transformer. The magnetic induction being practically independent of the load, the iron loss may be considered constant for

all loads and may be measured at no load. Since the exciting current is proportionally small, the I^2R loss may be neglected.

The copper losses may be computed from the current in each of the coils at the different loads. The resistance of the coils may be determined by the use of direct current and the fall of potential method. For accurate work the temperature of the coils should be taken at the time the resistance is measured and the resistance calculated for the proper operating temperature.

Another and usually a more accurate method of determining the copper loss is as follows. Connections are made as in Figure 33*B*, the low voltage coil being short circuited through an ammeter of the proper range. A low voltage at the proper frequency is then impressed upon the high voltage coil and regulated until the proper current flows in the ammeter. The reading of the wattmeter in the high voltage circuit gives the required copper loss, plus a small iron loss, and any additional losses occasioned by the current flowing. This small iron loss may be measured by applying the same low voltage to the primary, with the secondary on open circuit.

Having obtained data for the copper and iron losses, the commercial efficiency may be readily calculated from the relation

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{iron losses} + \text{copper losses}}.$$

The voltage being assumed constant, the output is proportional to the current. The iron losses are usually assumed constant. The copper losses may be found corresponding to the proper output current. If the regulation of the transformer is known, the corrected values of the secondary voltage may be used.

This method is not only convenient but also accurate since the losses, which are but a small percentage of the total power, are measured directly instead of measurements being made of the total output and total intake as in Experiment 32. An error as high as 10 percent. in the measurement of the losses would give a value of efficiency within 1 percent. even if the losses amounted

to 10 percent. of the total intake. In methods in which the intake and output are measured directly, an error of 10 percent. in the measurement of power would give the same 10 percent. error in the value of the efficiency.

If desirable, the eddy current losses in the conductors may be separated from the short circuit losses by the following method. The transformer is connected as in Figure 33*B*. A constant current is maintained on the secondary and the frequency of the supply voltage is varied over as wide a range as possible. A curve is plotted using watts as ordinates and frequency as abscissas. This curve, if extended backwards, will cut the watt axis at a point equal to the constant I^2R loss, corresponding to zero frequency, since the I^2R losses are constant and the eddy current losses vary with the frequency.

Data. Connections should be made as in Figure 33*A* and the core losses determined at normal voltage and frequency. Make connections as in Figure 33*B* and determine the copper losses at various values of secondary current from no load to about 50 percent. overload current. Measure the resistance of the transformer coils at some known temperature, one when the coils are heated from supplying a load, if possible.

Calculate. The value of efficiency for various loads between no load and 50 percent. overload. Calculate the all day efficiency of the transformer, assuming it to be operating 21 hours on open circuit and 3 hours at full load.

Curves. Plot a curve between copper losses and percent. load, using watts as ordinates. Plot a curve between efficiency and percent. load, using efficiency as ordinates.

Question. When the transformer is operating at 10 percent. of its normal pressure, why would the iron loss be much less than 10 percent. of its normal value?

No. 34. DETERMINATION OF THE REACTANCE DROP IN A TRANSFORMER UNDER LOAD.

References. Bedell, Chap. 7 and p. 388; Sever and Townsend, pp. 149 to 152; Karapetoff, pp. 444 to 450; Steinmetz, "A.C. Phenomena," pp. 249 to 252; Handbuch der Elektrotech., Vol. 2[4], pp. 219 to 222; Arnold, Vol. 2, pp. 282 to 284; Fleming, Vol. 2, pp. 595 to 598; see also references to Experiment 32.

Object. To determine the reactance drop caused by magnetic leakage in a transformer.

Theory and Method. Magnetic leakage in a transformer causes the same effect upon its regulation as a reactance connected in series with the primary. If the transformer is connected as in Figure 34*A* and low voltage alternating current

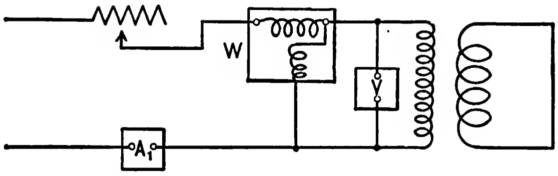

FIG. 34*A*. Connections for measuring the impedance drop in a transformer.

applied to the primary, the reactance drop may be calculated from the readings. The power as measured by the wattmeter consists of I^2R losses in the primary and secondary, plus a small iron loss, and eddy current loss in the conductors. The primary pressure has a power component IR, Figure 34*B*, sufficient to overcome these losses, and a wattless component, IX, due to the inductance caused by the leakage flux cutting the primary turns but not threading the secondary turns. Hence

$$IX = \sqrt{E^2 - \left(\frac{\text{Watts}}{I}\right)^2}$$

from which the vaiue of X may be computed. This quantity X is known as the equivalent, or leakage reactance of the transformer.

Data. Connect the transformer as in Figure 34*A*. Apply

different values of pressure to the primary so as to produce current in the secondary coils from no load to 50 percent. overload current. Take the readings indicated. If possible, apply full load current to the primary and vary the frequency over a wide range, keeping the current constant.

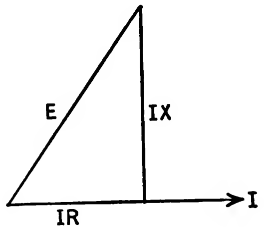

FIG. 34*B*. Impedance diagram of a transformer on short circuit.

Curves. Plot curves from the first set of readings, using watts, volts and reactance as ordinates and current as abscissas. Plot a curve from the second set of readings, using watts, volts and reactance as ordinates and frequency as abscissas.

Questions. How does the reactance drop vary with the frequency? How does the reactance drop affect the regulation?

No. 35. CALCULATION OF THE REGULATION OF A TRANSFORMER FROM NO LOAD OBSERVATIONS.

References. Bedell, Chap. 7 and p. 388; Sever and Townsend, pp. 149 to 152; Karapetoff, pp. 444 to 450; Steinmetz' "A.C. Phenomena," pp. 249 to 252; Handbuch der Elektrotech., Vol. 2[4], pp. 219 to 222; Arnold, Vol. 2, pp. 282 to 284; Fleming, Vol. 2, pp. 595 to 598; Thompson's "Dynamos," Vol. 2, pp. 601 and 602; see also references to Experiment 32.

Object. To determine the regulation of a transformer by calculation from no load observations.

Theory and Method. When a constant pressure is applied to the primary of a transformer, and the secondary is loaded, the change in secondary pressure depends upon the resistance and reactance drop in the transformer and upon the character of the load. The resistance and reactance drop may be determined as explained in Experiment 34. Regulation is defined as the ratio of the change in the secondary terminal voltage from full load

to no load, to the full load secondary voltage, the primary voltage being maintained constant.

Regulation may be determined by connecting the primary of the transformer to a circuit of the required voltage and frequency and loading the secondary. The change in terminal voltage is then observed from full load to no load, as in Experiment 36, the primary voltage being maintained constant. This method is not always feasible on account of the amount of power required for this test on large transformers, and much more reliance may be put on the results of calculation. The following method will be found simple and accurate.

For this method the data required are the open circuit core loss readings as measured in Experiment 33, the reactance as measured in Experiment 34, and the primary and secondary resistances.

The current in the primary is nearly in direct proportion to that in the secondary. If it be assumed directly proportional

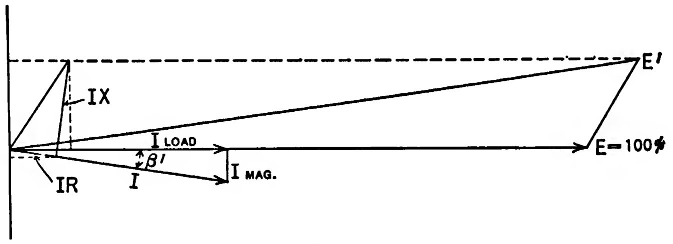

Fig. 35*A*. Electromotive force diagram of a transformer reduced to the primary side, non-inductive load.

and the magnetizing current added in its proper phase relation, the following equations may be written. Referring to Figure 35*A* and projecting the resistance and reactance drops upon the secondary voltage and a line at right angles through the origin as axes, it will be seen that

$$E' = \sqrt{(E + IR\cos\beta' + IX\sin\beta')^2 + (IX\cos\beta' - IR\sin\beta')^2} \quad (35a)$$

when all quantities are reduced to secondary values.

Since the magnetizing current is small in comparison with the

full load current, this may be written

$$E' = \sqrt{(E + IR)^2 + (IX)^2} \qquad (35b)$$

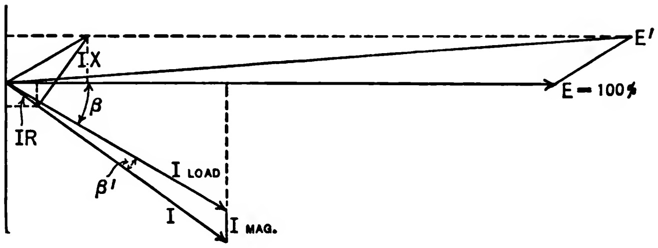

FIG. 35*B*. Electromotive force diagram of a transformer reduced to the primary side, inductive load.

From Figure 35*B*, for inductive load,

$$E' = \sqrt{[E + IR\cos(\beta + \beta') + IX\sin(\beta + \beta')]^2 + [IX\cos(\beta + \beta') - IR\sin(\beta + \beta')]^2}$$

or,

$$E' = \sqrt{[E + IR\cos\beta + IX\sin(\beta + \beta')]^2 + [IX\cos\beta - IR\sin(\beta + \beta')]^2}. \qquad (35c)$$

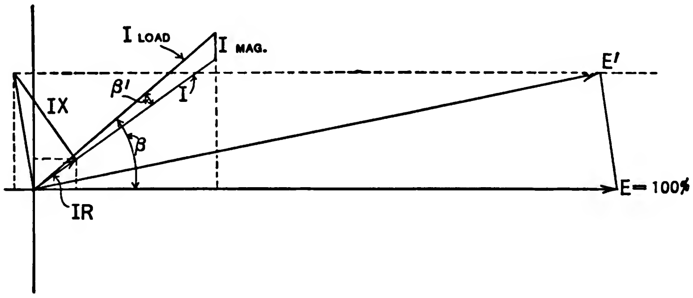

FIG. 35*C*. Electromotive force diagram of a transformer reduced to the primary side, capacity load.

From Figure 35*C*, for capacity load

$$E' = \sqrt{[E + IR\cos(\beta - \beta') - IX\sin(\beta - \beta')]^2 + [IX\cos(\beta - \beta') + IR\sin(\beta - \beta')]^2},$$

or

$$E' = \sqrt{[E + IR\cos\beta - IX\sin(\beta - \beta')]^2 + [IX\cos\beta + IR\sin(\beta - \beta')]^2}. \quad (35d)$$

These equations may be simplified in this application by the following method. The resistance and reactance drop and secondary voltage are all expressed in percent.

Then

IR = resistance drop in percent. of rated secondary voltage,
IX = reactance drop similarly expressed,
p = power factor of the load,
q = inductance factor of the load,
q' = wattless factor of primary current (or magnetizing current in percent. of full load current).
E' = secondary no load voltage in percent.,
E = secondary full load voltage equals 100 percent.

A further assumption is made that when one of the angles is small, the sine of their sum is equal to the sum of their sines.

Then, for non-inductive load,

$$E' = \sqrt{(100 + pIR + q'IX)^2 + (IX)^2}. \quad (35e)$$

For inductive load,

$$E' = \sqrt{[100 + pIR + (q + q')IX]^2 + [pIX - (q + q')IR]^2}. \quad (35f)$$

For capacity load,

$$E' = \sqrt{[100 + pIR - (q - q')IX]^2 + [pIX + (q - q')IR]^2}. \quad (35g)$$

The values of the various quantities are calculated in the manner shown in the following problem.

PROBLEM.

Calculation for a transformer of the following specifications:—

100 kw.,

15,000 volts primary,

2,350 volts secondary,
17.96 ohms primary resistance,
0.2103 ohm secondary resistance,
6.67 amperes primary,
42.5 amperes secondary.

From open circuit test, the following values are obtained:—

Core loss 1,220 watts, at 2,350 volts secondary,
Exciting current 0.725 ampere.

From short circuit test, the following values are obtained:—

Impedance volts 495,
Impedance watts 1,200,
Impedance amperes 6.67 (full load primary current),
Primary IR drop $= 6.67 \times 17.96 = 119.5 = 0.8$ percent. of 15,000.
Secondary IR drop $= 42.5 \times 0.2103 = 8.94 = 0.38$ percent. of 2,350.
Total IR drop $= 1.18$ percent.
Hence $IR = 1.18$.

Open circuit test,

Core loss watts $= 1,220$,
Exciting current $= 0.725$,

Energy current $= \dfrac{1,220}{2,350} = 0.522$,

Magnetizing current $= \sqrt{(0.725)^2 - (0.522)^2} = 0.505 =$ 1.19 percent. of 42.5 amperes, hence $q' = 0.0119$ (or say 0.012).

Impedance test,

Energy volts $= \dfrac{1,200}{6.67} = 180$,

Impedance volts $= 495$ (from test),

Reactance volts $= \sqrt{(495)^2 - (180)^2} = 460 = 3.07$ of 15,000, hence $IX = 3.07$.

REGULATION NON-INDUCTIVE LOAD.

$$p = 1.00, \quad q = 0, \quad q' = 0.012,$$

$$E' = \sqrt{(100 + 1.18 + 0.012 \times 3.07)^2 + (3.07)^2}$$
$$= 101.25.$$

Hence the regulation is 1.25 percent.

REGULATION INDUCTIVE LOAD, 80 PERCENT. POWER FACTOR.

$$p = 0.8, \quad q = 0.6, \quad q' = 0.012, \quad q + q' = 0.612,$$

$$E = \sqrt{(100 + 0.8 \times 1.18 + 0.612 \times 3.07)^2 + (0.8 \times 3.07 - 0.612 \times 1.18)^2}$$
$$= 102.89.$$

Hence the regulation is 2.89.

REGULATION CAPACITY LOAD, 80 PERCENT. POWER FACTOR.

$$p = 0.8, \quad q = 0.6, \quad q' = 0.012, \quad q - q' = 0.588,$$

$$E = \sqrt{(100 + 0.8 \times 1.18 - 0.588 \times 3.07)^2 + (0.8 \times 3.07 + 0.588 \times 1.18)^2}$$
$$= 99.19.$$

Hence the regulation is -0.81 percent.

Data. Obtain data as indicated in Experiments 33 and 34. Calculate the regulation of the transformer for non-inductive load and for inductive and capacity loads of 80 percent. power factor.

Questions. What would be the effect of constant load of variable power factor on the terminal voltage of the transformer? What effect would a low power factor have upon the secondary voltage? (Explain for both inductive and capacity loads.) How does the power factor of the load affect the rating and efficiency of the transformer? How is the regulation affected by a change in frequency, for the three classes of load?

No. 36. REGULATION OF A TRANSFORMER BY LOADING.

References. Bedell, pp. 174 to 186; Russell, Vol. 2, Chap. 11; Franklin and Esty, pp. 239 to 246; Handbuch der Elektrotech., Vol. 2^4, pp. 145 to 150; Arnold, Vol. 2, pp. 282 to 284 and 25 to 34; see also references to Experiments 34 and 35.

Object. To determine the regulation of a transformer under actual operating conditions.

Theory and Method. The potential at the secondary terminals of a transformer falls off from the value at no load when either non-inductive or inductive loads are applied. It may either fall or rise with a capacity load. This change in secondary voltage is due to the resistance of the two windings and to their self induction caused by magnetic leakage. This change in voltage is especially important when the transformers are connected to an incandescent lighting circuit. The change in voltage may be determined by loading the secondary of the transformer while the potential at the terminals of the primary is kept constant.

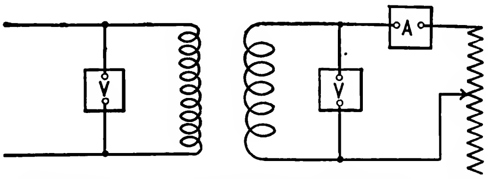

FIG. 36*A*. Connections for measuring the regulation of a transformer.

Connections should be made as shown in Figure 36*A* and a non-inductive load applied. Maintaining the primary pressure constant, the load should be varied from an overload to no load, readings of the instruments being taken. If the regulation on inductive or capacity load is also required, a wattmeter should be included in the secondary circuit and the power factor of the load held constant.

Another method of measuring the difference in potential due to the load is by the use of a second transformer and connections

as in Figure 36*B*. Load is applied to transformer No. 1. Transformer No. 2 is without load, but its secondary is connected in series with that of transformer No. 1, through a voltmeter V_2, the pressures of the secondaries of the two transformers being in

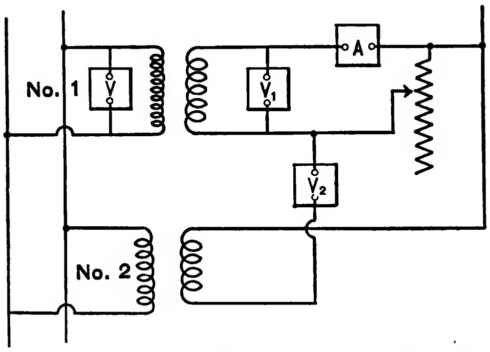

FIG. 36*B*. Connections for measuring the regulation of a transformer, modified method.

opposition. The reading of this voltmeter gives the drop in voltage due to the load on transformer No. 1. The difficulty with this method is that very low reading alternating current voltmeters are too delicate for use in ordinary laboratory service and are easily damaged. A step-up pressure transformer and a higher range instrument may be substituted for the low-range voltmeter V_2.

Data. Using one of the methods of connection described above, determine the regulation of a transformer for non-inductive load and for inductive and capacity loads of 80 percent. power factor.

Curves. Plot curves with secondary voltage as ordinates and percent. load as abscissas, for each condition of loading.

Question. What effect does a difference in regulation have on the proportion of load upon each of two transformers connected in multiple.

No. 37. REGULATION OF A THREE-WIRE TRANSFORMER.

References. Bedell, pp. 13 and 14; Arnold, Vol. 2, pp. 61 to 67 and 302 to 303; see also references to Experiments 34, 35 and 36.

Object. To study the regulation of a three-wire transformer under various kinds of loads.

Theory and Method. The three-wire system of distribution gives such an advantage in economy over a two-wire system where any considerable amount of power is required at low voltage, that some method must be used to obtain a well balanced system of this kind. Two transformers may be used to feed such a system, or the two secondaries of a single transformer may be connected in series, the neutral being connected to their point of junction. The use of a single transformer gives a higher efficiency and such a system will be considered here.

The cross section of the neutral may be taken equal to or half that of each of the outside wires, particularly if the load is non-inductive. The data should show why, with mixed lighting and motor loads between neutral and either of the outside wires, the neutral wire should have the same cross section as the other two wires.

Data. Connect the transformer as shown in Figure 37. Keeping the frequency and the primary voltage constant at their

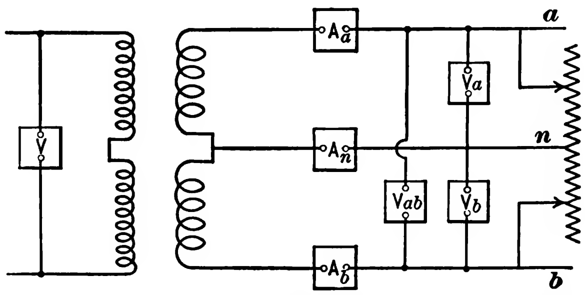

FIG. 37. Connections for measuring the regulation of a three-wire transformer.

normal values, observe E_a, E_b, E_{ab}, I_a, I_b and I_n, under the following conditions:—

1. Gradually increase the current in sides (a) and (b) by equal increments from no load to full load value, the load being non-inductive.

2. Throw off some of the load on (a) and gradually replace it with an inductive load of a value which will make I_a and I_b equal, taking readings of each increment of I_a.

3. Remove the inductive load and, after removing a sufficient part of the load on (b) to make I_a and I_b equal, place an inductive load between (a) and (b). This inductive load should be sufficient to bring the currents I_a and I_b to their full load value.

4*a*. Starting with both sides unloaded, gradually load side (a) with a non-inductive load to one half of the full load current, taking observations for each increment of I_a.

4*b*. Without varying the conditions on side (a), gradually apply a non-inductive load on side (b) until full load current is obtained, taking readings of each increment of I_b.

4*c*. Maintaining the resistance of the load on (b) constant, gradually increase the load on (a) until I_a and I_b are equal, taking readings of each increment of I_a.

Curves. With pressure as ordinates and current as abscissas, plot the following curves:

A. From conditions (1) and (2),

$$E_{ab},\ E_a \text{ and } E_b \text{ with } \tfrac{1}{2}(I_a + I_b).$$

B. From condition (4),

$$E_{ab} \text{ with } \tfrac{1}{2}(I_a + I_b),\quad E_a \text{ with } I_a \text{ and } E_b \text{ with } I_b.$$

In discussing these curves, note carefully their shape as regulation curves and explain any peculiarities. In (B) be sure to show E_a and E_b on the same axes so that their relative variations may be more apparent. Indicate the parts of these curves taken from each set of data, as (4*a*), (4*b*) and (4*c*).

Questions. What effect would a capacity have in cases (2)

and (3)? Why is the neutral current not equal to $I_a - I_b$ in case (2)? What advantage has the direct current three-wire system over the alternating current system from the standpoint of regulation?

A core type step down transformer has two primary and two secondary coils. The primaries are connected in series. Why might it be possible to short circuit one of the secondary coils without blowing the fuses in the primary side?

Would this be possible in the case of a shell type transformer? What precautions may be taken in winding transformers for three-wire service, in order to insure good regulation? How should motors and arc lamps be connected when operating on this system? If one of the secondary coils is reversed, what conditions will exist?

No. 38. STUDY OF A CONSTANT CURRENT TRANSFORMER.

References. Bedell, Chap. 8, p. 226; see also references to Experiments 34, 35 and 36.

Object. To study the behavior of a transformer supplied with a constant current in its primary and a variable impedance in its secondary. This is the condition of operation of one of the early systems of arc lighting.

Theory and Method. From the fundamental equation, the flux required in the case of a given transformer depends upon the electromotive force to be generated, or

$$\phi = \frac{E \times 10^8}{4.44fn}.$$

In a constant current transformer the value of E depends upon the impedance drop in the secondary circuit Except for the magnetic leakage, the primary electromotive force is in direct ratio to the secondary electromotive force. The current in the primary is made up of two factors, that required to produce the flux ϕ and that required to balance the secondary current I_2.

The portion to produce the flux ϕ depends upon the permeability of the iron and the value of ϕ for a given winding. Since the primary current is constant, the portion to balance the secondary current is the vector difference between the total primary current and the exciting current. Hence the greater the value of secondary electromotive force, the greater the value of exciting current and the greater the difference from the true ratio of transformation. If the iron approach saturation, the total current in the primary may be used to produce the flux and the secondary current will be zero. This is the case for open circuit of the secondary. A value of secondary voltage must then exist for a given power factor of the load, after which the secondary current falls off rapidly with increase of secondary impedance. A capacity load in the secondary would produce different results because part, or all of the exciting current, would then be supplied by the leading current.

The copper losses remain practically constant throughout the useful range of regulation. The iron losses increase with the primary voltage, since the flux also increases. At short circuit the copper losses are a maximum. At open circuit the copper loss in the secondary is zero and in the primary normal, while the iron loss is a maximum. The point of maximum output occurs when the secondary current begins to fall off more rapidly than the secondary electromotive force increases.

A source of constant primary current may be obtained from a constant current alternator, a constant pressure to constant current transformer or the current may be kept constant by adjustment of the primary circuit. The secondary should be connected to a variable non-inductive load, and current and electromotive force read. The power, current and electromotive force in the primary should be taken. Load should be varied from short circuit to open circuit, if possible. The efficiency and regulation should be calculated from these readings.

The iron loss corresponding to each voltage may be determined

with the secondary on open circuit. The copper loss may be determined from the resistances of the two windings.

Suggestion. A transformer of low ratio should be used, to avoid high voltages.

Data. Take the necessary observations for regulation, efficiency and power factor as indicated above. Determine the iron and copper losses for a range covering the conditions of operation.

Curves. Plot the following curves, using secondary output as abscissas:—

1. Efficiency.
2. Secondary current.
3. Power factor.
4. Primary electromotive force.
5. Secondary electromotive force.
6. Iron loss.
7. Copper loss.

Question. Suppose a constant pressure transformer and a constant current transformer have the same output, the same full load efficiency and the same value of contant losses; what will be the general shape of their efficiency curves below full load?

No. 39. REGULATION OF A CURRENT TRANSFORMER.

References. Esty, pp. 216 to 218 and 253 to 254; Franklin and Esty, pp. 46 and 234; Standard Handbook, Sec. 6, Art. 142 to 145; Handbuch der Elektrotech., Vol. 2^4, pp. 20 and 21; Arnold, Vol. 2, p. 317; *Trans. Am. Inst. Elec. Eng.*, September, 1906, K. L. Curtis, The current transformer; *Elec. Wld.*, September 1, 1906, W. B. Gump, Properties of the series transformer; May 16, 1908, E. S. Harrar, The series transformer.

Object. To determine the range of accuracy of a current transformer when supplied with a variable current at constant frequency.

Theory and Method. If a transformer is connected with its primary in series with one of the line wires of an alternating current circuit, and with its secondary across the terminals of a non-inductive resistance, the secondary current will be $I_2 = kI_1$. This relation will be maintained throughout a considerable range of the primary current, providing the magnetic leakage is negligible and that the iron of the core is worked on that portion of its magnetization curve for which the permeability is practically constant. Unlike a constant potential transformer, the resistance of the secondary circuit is maintained constant. The regulation is the ratio of the primary current (multiplied by the ratio of transformation) to the observed secondary current. It is that value by which the stated ratio must be multiplied to obtain the ratio found by observation. The primary pressure of the transformer will be

$$E_1 = I_1\sqrt{(k^2R_2 + R_1)^2 + (k^2X_2 + X_1)^2},$$

R_2 and X_2 being the total resistance and reactance, respectively, of the secondary circuit. Increasing the secondary resistance decreases the range of good regulation of the transformer, because the primary pressure will sooner reach the point at which the change of permeability becomes appreciable. The power factor of the load, the current of which passes through the primary, does not affect the operation of the transformer.

Current transformers are used largely as step-down transformers in connection with such devices as ammeters, wattmeters and circuit breakers; the object being to reduce the size of the wire in the current coils of the instruments and, in the case of high pressure circuits, to avoid their presence within the instruments. The secondary current is displaced nearly 180 degrees in phase, with respect to the primary current. Consequently, wattmeters will register correctly (considering the ratio of transformation) no matter what the phase relation between the current and pressure in the main line, provided they are properly connected to the current transformer. Another important ap-

plication may be found in the compensated alternator in connection with the rectifying brushes. For obvious reasons, the total impedance of a current transformer should be small and, as its application is one requiring but little power, this ideal condition can be easily met.

From what has been said of the factors affecting regulation, it is seen that the impedance of the current coils of instruments used with current transformers should be negligible, otherwise each instrument and its transformer should be calibrated as a unit.

Data. In the present test a current transformer is connected to a line and to an ammeter. The primary and secondary currents are observed from zero value of the primary current to a value considerably beyond the point where the secondary current departs from its theoretical value, $(I_2 = kI_1)$. Tests should be made under the following conditions:—

1. With a non-inductive load on the line.
2. A resistance equal to about twice the total normal resistance of the secondary circuit is then inserted in series with the secondary, and similar observations taken.
3. This resistance being removed, a small impedance coil is inserted in its place and the observations again taken.
4. Open circuit the secondary and read pressure across primary and secondary at normal current.

Curves. Using primary currents as abscissas, curves should be plotted showing the values of the secondary current, in percent. of the theoretical values, for each of the conditions of the experiment.

Questions. A current transformer is connected in the usual manner with the secondary short circuited through an ammeter. The meter circuit becomes broken. What will happen to the transformer? Will there be any danger in handling the secondary connections when alive? What will be the effect on the line voltage? On high potential circuits the secondary is often grounded on one side. Why?

No. 40. OPERATION OF CONSTANT POTENTIAL TO CONSTANT CURRENT TRANSFORMER.

References. Steinmetz' "A.C. Phenomena," pp. 85 and 119; Esty, pp. 255 to 260; Franklin and Esty, pp. 215 to 217; Thompson's "Dynamos," Vol. 2, p. 603; Standard Handbook, Sec. 6, Art. 146 to 149; Arnold, Vol. 2, pp. 329 to 333.

Object. To study the behavior of a type of transformer used for operating series arc lights when connected to a circuit of constant potential.

Theory and Method. Figure 40 shows the general form of a transformer commonly used for series arc light circuits. In this type of transformer a constant potential is impressed upon a stationary primary winding. The secondary winding is free to move longitudinally along the center core of the shell type transformer. When current is taken from this secondary winding a repulsion takes place between the two coils. This causes the secondary coil to take a position where the repulsive effect just balances the excess weight of the coil.

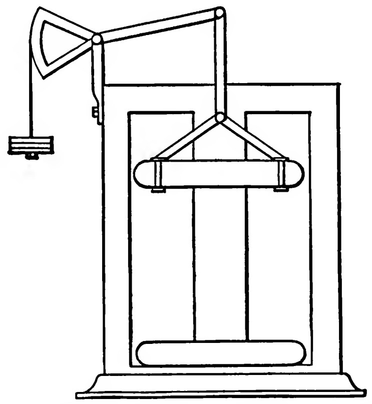

FIG. 40. Diagrammatic representation of an arc light transformer.

There is a leakage flux between the sides and the center core of iron. Part of the lines set up by the primary pass through the secondary coil. These latter lines induce an electromotive force

in the secondary coil sufficient to send the required current through the external circuit. The closer the secondary coil approaches the primary the less the number of leakage lines and the greater the induced secondary voltage. This corresponds to the position required when there are many lights connected. As lights are turned off, the current tends to increase and the repulsive action sends the coils apart. This reduces the secondary potential and tends to maintain constant current in the secondary circuit. The action of the coil is entirely automatic and the current is kept nearly constant in the secondary circuit.

The primary winding must supply current to magnetize the core of the transformer and to supply the losses, even when there is no current taken from the secondary. The power factor of the secondary circuit is usually about 80 percent. with the ordinary arc lamps. The leakage flux between primary and secondary of the transformer causes a quadrature current. The power factor of such a transformer is, in consequence, rather low at all loads.

Caution. It must be remembered that the potentials on both the primary and the secondary are usually dangerous. Extreme care must be taken in making tests on a transformer of this type to protect the person from accidental contact with live wires.

Data. Operate the transformer with constant potential on its primary. Connect the secondary to a non-inductive resistance. Vary the resistance from short circuit to open circuit, allowing the secondary coil to take its natural position, and with the counter weights properly adjusted for normal operation. Take readings of primary current and power, and of secondary current and electromotive force. Repeat, using for a load constant current arc lamps, varying the load one lamp at a time, and taking readings as before, including a wattmeter in the secondary circuit. Remember in each case to have the pressure coil of the wattmeter of sufficient range to stand the full potential, or use a potential transformer with the wattmeter and voltmeters. Be careful in handling the ammeters, as full potential is on their terminals unless they are protected by current transformers.

Curves. Using watts output as abscissas, plot curves of the following quantities for each of the above conditions of loading:—

1. Efficiency.
2. Primary power factor.
3. Secondary power factor.
4. Primary current.
5. Secondary current.
6. Primary watts.
7. Secondary pressure.

Question. Why do the iron losses increase slightly with the load?

Discussion. Discuss the several losses, showing how they vary with the load and how they affect the form of the efficiency curve.

No. 41. SEPARATION OF THE HYSTERESIS AND EDDY CURRENT LOSSES IN THE CORE OF A TRANSFORMER.

References. Russell, Vol. 2, pp. 308 to 310; Karapetoff, p. 227; Arnold, Vol. 2, pp. 272 to 280; *Elec. Wld.*, Vol. 31, p. 306. See also references to Experiment 31.

Object. To separate the total core losses of a transformer into hysteresis and eddy current losses.

Theory and Method. It is often desirable to separate the two quantities entering into the core loss of a transformer. This question especially interests the manufacturer and the designer.

By adding together Equations (31*a*) and (31*b*) of Experiment 31, the following equation is derived:—

$$P = P_h + P_f = k_h f \mathcal{B}^{1.6} + k_f f^2 \mathcal{B}^2. \qquad (41a)$$

With flux density constant, this may be written

$$P = k_h' f + k_f' f^2. \qquad (41b)$$

By dividing this equation through by (f), we have

$$\frac{P}{f} = k_h' + k_f'f,$$

which is the equation of a straight line. If a curve be plotted with the first member as ordinates and with frequency as abscissas, this curve will, if extended, cut the axis of ordinates at a distance k_h' above the origin, thus determining its value as may be seen in Figure 41*A*. This value multiplied by normal frequency gives the value of the hysteresis loss at that frequency and the value of flux density corresponding to normal voltage.

A curve may also be plotted from Equation (41*b*) between total watts and frequency. The tangent of this curve at the point of

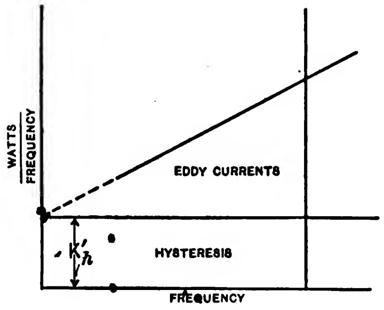

FIG. 41*A*. Curves for separation of the hysteresis and eddy current losses in the core of a transformer.

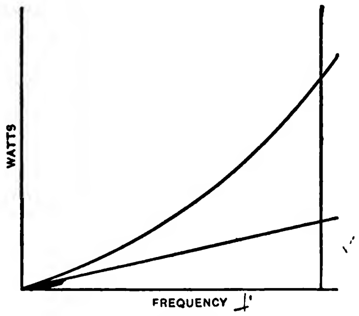

FIG. 41*B*. Curves for separation of the hysteresis and eddy current losses in the core of a transformer.

intersection with the watt axis divides the ordinate at normal frequency into two parts equal to the hysteresis and eddy current loss as shown in Figure 41*B*. The first method is the more accurate.

Where it is impossible to control the frequency of the source of alternating current over a wide range of values, the following method may be used. It is usually possible to obtain an alternating current at a fairly constant frequency. The pressure may be varied by any one of several different methods. Measure the loss at two pressures differing from each other by a sufficient

amount. Let the subscript (1) refer to the first pressure and the subscript (2) to the second pressure; then

$$P_1 = k_h''E_1^{1.6} + k_f''E_1^2,$$

and

$$P_2 = k_h''E_2^{1.6} + k_f''E_2^2.$$

These two equations with two unknowns may be solved readily. These values may then be substituted and values of the separate quantities determined.

Data. If Experiments 30 and 31 have been performed, the data may be used for the separation of the losses. If not, data may be obtained for conditions (*a*), (*b*) and (*d*) as explained in Experiment 30. Care should be taken to adjust for normal flux density.

Curves. Plot curves similar to those in Figures 41*A* and 41*B*. From these curves separate the core losses at normal frequency and pressure.

Questions. Have you any suggestions for improvements in the design or construction of the transformer? Suppose that some alloy having a higher electrical resistance than that of iron, is used for the core; would the eddy currents be increased or diminished?

No. 42. DETERMINATION OF STEINMETZ' EXPONENT AND COEFFICIENT.

References. Russell, Vol. 2, pp. 308 to 310; Karapetoff, p. 227; Arnold, Vol. 2, pp. 272 to 280; *Elec. Wld.*, Vol. 31, p. 306; see also references to Experiment 31.

Object. To determine the value of Steinmetz' exponent and coefficient for the iron in a transformer.

Theory and Method. The law of hysteresis loss in iron, frequency being constant, is expressed by the formula

$$\begin{aligned} P_h &= P - P_e, \\ &= k_h \mathcal{B}^x, \\ &= k_h' E^x. \end{aligned} \tag{42a}$$

Hence,

$$\log P_h = x \log E + \log k_h'. \qquad (42b)$$

This is the equation of a straight line, the tangent of the angle of inclination of which to the axis of log E, is the value of x. This quantity (x) is known as Steinmetz' exponent for iron. It is found to be approximately 1.6 over the ordinary range of flux density. To obtain this value a curve should be plotted with log E as abscissas and log P_h as ordinates. The inclination of this line to the base should be about 58 degrees.

When the formula is expressed so as to give the loss in the iron in ergs per cubic centimeter per cycle, as

$$P_h = \eta \mathcal{B}^x,$$

the coefficient η which depends upon the electrical qualities of the iron, may be determined. This coefficient is contained in the value of k_h' as derived above. In order to obtain the value of η, known as Steinmetz' coefficient, the dimensions of the iron core must also be known. It is customary to take the value of x as 1.6 in the calculation of this coefficient.

Data. Using data derived as in Experiment 31, separate the losses as described in Experiment 41. The eddy current losses having been determined for normal pressure and frequency, the value of the eddy current loss for any other pressure at the same frequency may be derived, by proportion. This may be subtracted from the total core loss at that pressure and the value of the hysteresis loss derived. Measure the dimensions of the transformer core.

Curve. Plot a curve between log E and log P_h, using log P_h as ordinates. Determine the value of the exponent and coefficient as explained.

Question. Is the iron of a good quality?

No. 43. EFFICIENCY TEST OF TRANSFORMERS BY THE OPPOSITION METHOD.

References. Bedell, p. 377; Fleming, Vol. 2, p. 602; Sever and Townsend, p 154; Handbuch der Elektrotech., Vol. 2[4], pp. 215 to 217; Arnold, Vol. 2, pp. 284 to 288; *Bull. Univ. Wis.*, Vol. 1, No. 11, A. H Ford; *Elect'n Lond.*, Vol. 29, 1892, pp. 223 and 665; *Elec. Wld.*, October 8, 1892.

Object. To measure the efficiency of a transformer under normal operating conditions as to copper and iron losses, and at the same time to dissipate only that amount of energy required to furnish the losses

Theory and Method. The primaries of two like transformers T_1 and T_2, Figure 43*A*, are connected to a supply circuit in such a manner that the secondaries, when mutually short cir-

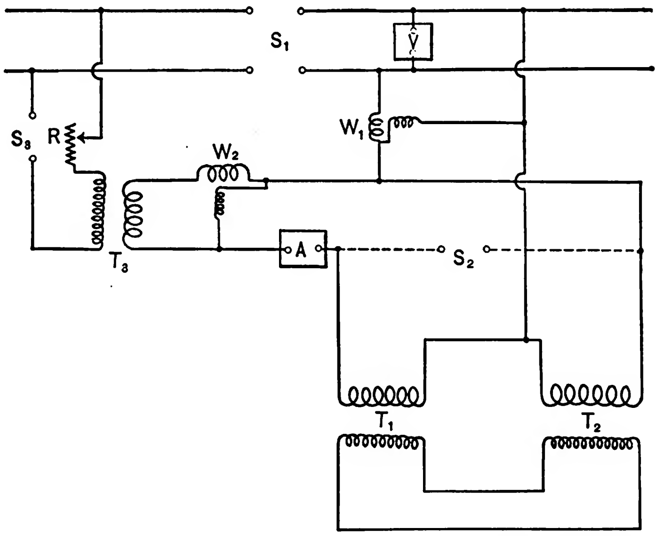

FIG. 43*A*. Connections for measuring the efficiency of two transformers by the opposition method.

cuited, will be in opposition. A third transformer T_3 is connected with its secondary in series with the primary circuit of T_1. A variable resistance R is inserted in the primary circuit

of T_3. The secondary pressure of T_3 will be added to or subtracted from that impressed upon T_1 and a current will flow through both the primary and the secondary coils of T_1 and T_2. By adjusting R, any desired current may be caused to flow. The indications of wattmeter W_1 will closely approximate the iron losses of the transformers T_1 and T_2 while that of the wattmeter W_2 (the output of T_3) will give the copper losses in the two transformers plus the I^2R loss in the leads, contacts and ammeter A. A correction for these external losses is easily made by closing the switch S_2 and adjusting the current to the proper amount. Let the readings of W_2 under this condition be represented by P_c. Assuming a non-inductive load, the efficiency of either of the transformers under test will be for the current I and the voltage E,

$$\text{Efficiency} = \frac{IE}{IE + \dfrac{P_1 + P_2 - P_c}{2}},$$

IE being the output. Taking IE as intake,

$$\text{Efficiency} = \frac{IE - \dfrac{P_1 + P_2 - P_c}{2}}{IE}.$$

It is preferable, in making connections, to mutually short circuit the high pressure coils of T_1 and T_2, all measurements being made with low tension. The auxilliary transformer must be capable of withstanding the full load primary current of either T_1 or T_2, although its output need not be greater than the measured copper losses. A special ratio transformer is desirable for this purpose.

Caution. Before mutually short circuiting T_1 and T_2, their secondary pressures should be tested by a direct current voltmeter for polarity to insure opposition. Another method is to close S_3, leaving S_1 open. If opposition has not been established, no appreciable current can be made to flow.

Modification. It is sometimes desirable to connect T_3 in the secondary circuit, as shown in Figure 43*B*. Here the test for

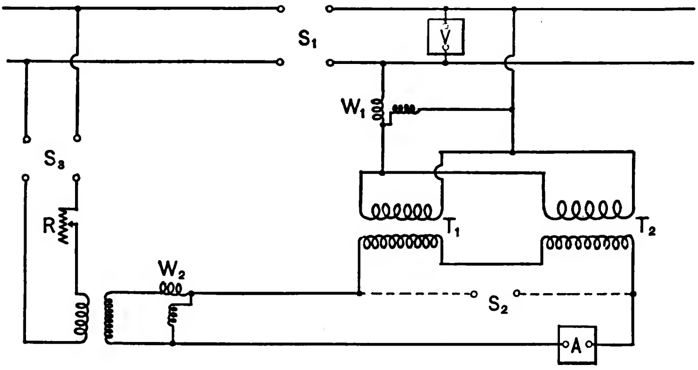

FIG. 43*B*. Connections for measuring the efficiency of two transformers by the opposition method.

opposition may be made as before by closing S_3. In this case the efficiency is calculated from measurements on the secondary side of T_1 and T_2.

Both the above methods are open to the objection that two transformers having exactly equal losses are necessary to insure accuracy. This difficulty is surmountable by a somewhat complicated procedure, as follows. Three transformers (*a*), (*b*) and (*c*), of the same capacity and voltage may be tested in three pairs and their losses separated by algebraic methods. As an example, let the iron loss $(P_a + P_b)$ for (*a*) and (*b*) be P_{ab}, that for (*b*) and (*c*), P_{bc}, etc. Then

$$\begin{array}{l} P_a + P_b = P_{ab} \\ P_a + P_c = P_{ac} \\ \hline P_b - P_c = P_{ab} - P_{ac} \end{array}$$

but

$$P_b + P_c = P_{bc}.$$

Therefore

$$P_b = \frac{P_{ab} + P_{bc} - P_{ac}}{2}.$$

Where a large number of transformers are tested, a standard transformer whose losses have been previously determined by the stray power method, may be used as, say T_2, and each trans-

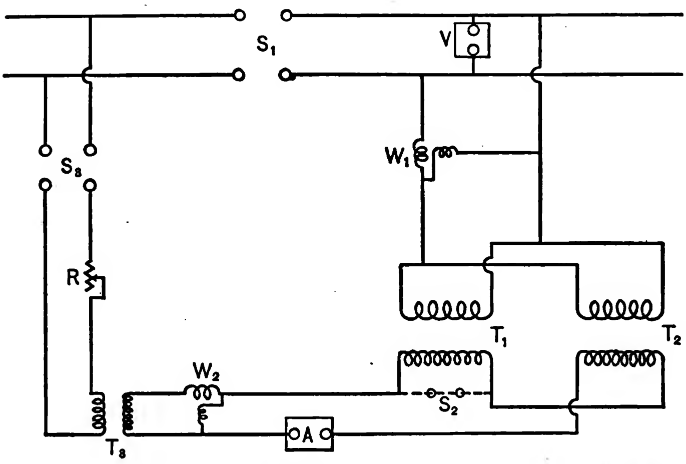

FIG. 43*C*. Connections for measuring the efficiency of one transformer by the opposition method.

former to be tested used in turn as T_1. Each loss is then easily determined by subtracting the known loss from the proper wattmeter readings, A fair degree of accuracy is possible even with

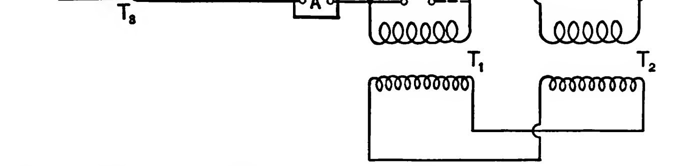

FIG. 43*D*. Connections for measuring the efficiency of one transformer by the opposition method.

considerable inequality of losses. The standard transformers need not be of the same capacity but should never be overloaded greatly. This method, like the other, has the advantage that heating, regulation and efficiency tests may be made without a change of connections.

When the transformer has two primaries and two secondaries, several modifications of this method, using only the one transformer and an auxiliary, may be applied. Two such modifications are shown in Figures 43*C* and 43*D*. In the case of Figure 43*D*, the switch S_2 should never be closed while the iron losses are being supplied.

Data. Using one of the methods described, obtain data for calculating the efficiency of a transformer at various loads.

Curve. Plot an efficiency curve, using percent. load as abscissas.

No. 44. TEMPERATURE TEST OF A TRANSFORMER.

References. Karapetoff, pp. 440 to 442; Sever and Townsend, p. 154; Fleming, Vol. 2, p. 599; Standard Handbook, Sec. 6, Art. 102 and 103; Arnold, Vol. 2, pp. 288 to 291; Standardization Rules Am. Inst. Elec. Eng.

Object. To determine the temperature rise in a transformer when operated under a given condition as to loading.

Theory and Method. The temperature of a transformer connected to a source of potential will be higher than that of the room in which it is placed, due to the heat produced by the losses in the core and copper. Although the efficiency of a transformer is higher than that of most other electrical apparatus and hence the heat to be dissipated is less for a given capacity, there are no moving parts to aid in its dissipation. Small transformers are cooled by radiation from the outside of the containing cases. In large sizes it is necessary to resort to artificial cooling. Excessive temperature in the transformer may cause deterioration of the insulation and a consequent burn out of the coils.

In transformers properly rated for continuous service, the maximum temperature will be reached in from three to eight hours operation at full load, depending upon the size of the unit. The radiating surface of a transformer increases directly with the square of the linear dimensions, while the capacity (and hence, approximately, the losses) increases roughly as the cube of the same dimensions. Taking a standard room temperature of 25° C., a permissible rise in the coils determined by resistance measurements is 50° C. The same rise determined by thermometer is permissible in the other parts. The temperature rise by resistance is usually the more reliable.

The transformer should be operated at normal voltage and frequency and at full load. Temperature readings should be taken at short intervals (quarter or half hour) depending upon the size of the unit, until a constant temperature is reached. The room thermometers should be screened from draughts and thermal radiation. If the transformer is artificially cooled by air or liquid, the test should be made under normal conditions. The amount of cooling agent should be determined. The temperature of the cooling agent, before entering and after passing through the transformer, should be measured. The resistances of the windings should be taken before starting, with the transformer at room temperature. Resistance readings of the windings should be taken at intervals during the test and immediately at the close of the test, before the transformer has had time to cool down appreciably, and a set of thermometer readings should also be taken at each of these intervals. From these the temperature rise may be computed by the following formula:—

$$R_t = R(1 + 0.0042\theta),$$

where

R = the resistance at room temperature,

R_t = the resistance at required temperature,

θ = the temperature rise in degrees centigrade.

The rise in temperature should always be corrected to a standard room temperature of 25° C. This may be done by increas-

ing or decreasing the observed rise θ one half percent. for each degree of room temperature below or above the standard of 25° centigrade. If the thermometer gives a greater rise in temperature than the calculated value, the thermometer reading should be chosen.

The most economical method of loading the transformer is by one of the methods described in Experiment 43. If more convenient, a non-inductive load may be used.

Data. Measure the resistances of the windings before starting the test, with the transformer at room temperature. Operate the transformer at full load under normal conditions of voltage and frequency. Take temperature readings at frequent intervals (quarter or half hour, depending upon the size of the unit) until a constant temperature is reached. Determine the amount of cooling agent, if any; also measure its temperature before entering and after passing through the transformer. Measure the resistances of the windings immediately after the completion of the run and take a final set of thermometer measurements.

Compute. The rise in temperature from the resistance measurements and compare this value with that obtained by means of the thermometers. Correct the temperature readings to a standard room temperature of 25° C.

Curves. Using time as abscissas, plot curves showing the corrected rise in temperature for each part of the transformer.

Questions. Why is the room temperature reduced to a standard? What correction would you apply to temperature readings at several thousand feet above sea level?

No. 45. REGULATION, EFFICIENCY AND POWER FACTOR OF A TRANSFORMER SYSTEM INVOLVING TWO OR MORE PRESSURE TRANSFORMATIONS.

References. Bedell, Chap. 7, pp. 129 to 131; *Can. Elec. News,* November, 1905, R. T. Mackeen, The multiple operation of trans-

formers; *Elec. Times,* January 16, 1902, A. G. Hansard, Paralleling of alternating current transformers; *Elec. Wld.,* July 18, 1908; E. G. Reed, Parallel operation of transformers; see also references to Experiment 34.

Object. To study a transformer system involving two or more pressure transformations.

Theory and Method. A common arrangement of transformers for long distance transmission is shown in Figure 45. Low voltage is generated at the power house and it is then stepped up to a high pressure to economize on copper in the transmission line. At the substation it is stepped down to a reasonable pressure for distribution and is again reduced for local uses. This involves a series of three pressure transformations.

When a self-inductance L is placed in the secondary of a transformer it is equivalent to a self-inductance of k^2L in the primary circuit (where k is the ratio of transformation), and similarly, a resistance R in the secondary circuit is equivalent to k^2R in the primary.

Knowing the resistance of the coils of a transformer, its reactive drop, its magnetizing currents and the core losses, the total drop for any current can be calculated. The magnetizing current is usually negligible. If an external inductance or capacity is inserted in the secondary circuit, the drop due to it can be calculated in terms of the primary pressure, by multiplying by the square of the ratio of transformation.

Referring to the diagram, the practical problem is to find the generator pressure required to deliver a given pressure E at the receivers, the load and power factor at this point being known.

First, add to the reactive drop of the primary of transformer C, the reactive drop of the load, reduced to primary pressure. This combined vectorially with the total IR drop of the transformer and primary pressure required to deliver E volts on open circuit will give the required primary pressure. Taking this last pressure as that delivered by transformer B, its primary

pressure can be similarly obtained, and so on. Any line losses, inductances or capacities should be considered in the order of their meeting. The final pressure to be added vectorially to that calculated by using ratios of transformation only, is the sum of the *IR* drops reduced to generator pressure added vectorially to the sum of the reactances (inductive and capacity), reduced

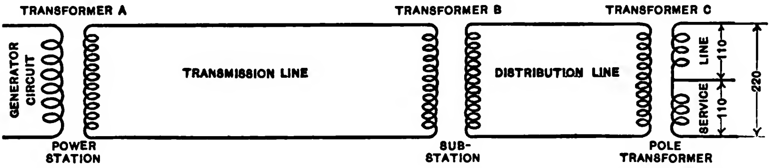

Fig. 45. Connections for measuring the regulation and efficiency of a system containing two or more transformations.

to generator pressure; but for accurate results it is necessary to consider each transformer separately, taking into account its secondary pressure and its losses.

Data. Connect up two or more transformers of about equal capacity as in Figure 45 and give the last one a load, the power factor being about 80 percent. at full load current. Measure the power and power factor of the load and the power factor and intake of the system. Be certain of the frequency. Measure all the voltages and currents at the transformer terminals and be certain to take into account the resistances of the leads if of appreciable magnitude. Measure the resistances of the individual transformers, their inductive drops at the measured currents, and core losses at the measured voltages.

Calculate. The combined efficiency, combined regulation and combined power factor.

Question. Explain how an inductive load of given power factor affects the efficiency and regulation of the system as compared with the same load at a higher power factor.

No. 46. THE TRANSFORMER AS A POTENTIAL REGULATOR.

References. Bedell, p. 326; Thompson's "Dynamos," Vol. 2, p. 605; Esty, pp. 392 to 399; Russell, Vol. 2, pp. 282 to 287; Standard Handbook, Sec. 6, Art. 153 to 161; Arnold, Vol. 2, pp. 321 to 329; *Trans. Am. Inst. Elec. Eng.*, Vol. 12, p. 549, W. L. R. Emmet, Pressure regulation; *Elec. Wld.*, Vol. 27, p. 549.

Object. To test a potential regulator.

Theory and Method. The flexibility of the transformer is well shown here in a practical application. Advantage is taken of the fact that, on a non-inductive load, the secondary pressure is practically at a phase difference of 180 degrees from the primary pressure. It can thus be either added to or subtracted from the primary pressure by making the proper connections. Figure 46*A* indicates a method of connecting the secondary to

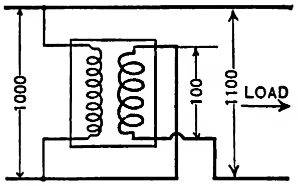

FIG. 46*A*. Connections of a transformer to raise the pressure.

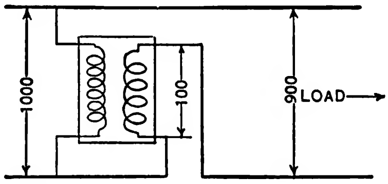

FIG. 46*B*. Connections of a transformer to lower the pressure.

raise the pressure of the mains and Figure 46*B* shows the connections for lowering the pressure. Figure 46*C* illustrates a method for either raising or lowering the pressure by increments. Here the secondary coil is tapped out at intervals, the taps being connected to stationary contact lugs. The two movable contacts *A* and *B* are geared together to move equal amounts in opposite directions, the line being connected to the two arcs. When *A* and *B* rest on the center contact the pressure is neither raised nor lowered.

Another method of accomplishing the same result is shown

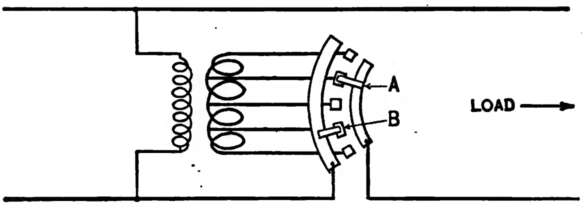

FIG. 46*C*. Diagrammatic scheme for raising or lowering the pressure by increments.

in Figure 46*D*. This is known as the Stillwell regulator. The pressure is added to or subtracted from that of the generator according to the position of the primary reversing switch. The preventive resistance shown in the diagram is used to avoid a short circuit on one coil of the transformer during a transition of the pressure. It also suppresses arcing.

In the diagrams so far shown the secondary alone is subdivided, but it is evident that commutation of the primary turns would be equally effective. In general, it is advisable to commutate the low pressure coil, whether it is primary or secondary, thus minimizing the tendency toward arcing at the contacts. In the case of extremely low pressures and heavy currents, it is more economical from the standpoint of construction and maintenance to commutate the high pressure coil, thus avoiding the use of massive contacts.

Where a fine gradation of pressure over a wide range is required, a multiplicity of contacts is avoided by subdividing the variable coils into a small number of equal parts and again subdividing one of these small sections. When this section is commutated over its entire range an equivalent section is automatically substituted and the first section placed in series ready for commutation again. The potential regulators illustrated in Figures 46*A*, 46*B*, 46*C* and 46*D* are known as compensator potential regulators.

The ideal method of regulating the secondary pressure in a transformer of this kind is by a smooth change. This may be accomplished by changing the flux which threads the secondary.

Such a transformer, which is known as a magneto-potential regulator, may be made to raise or lower the pressure by chang-

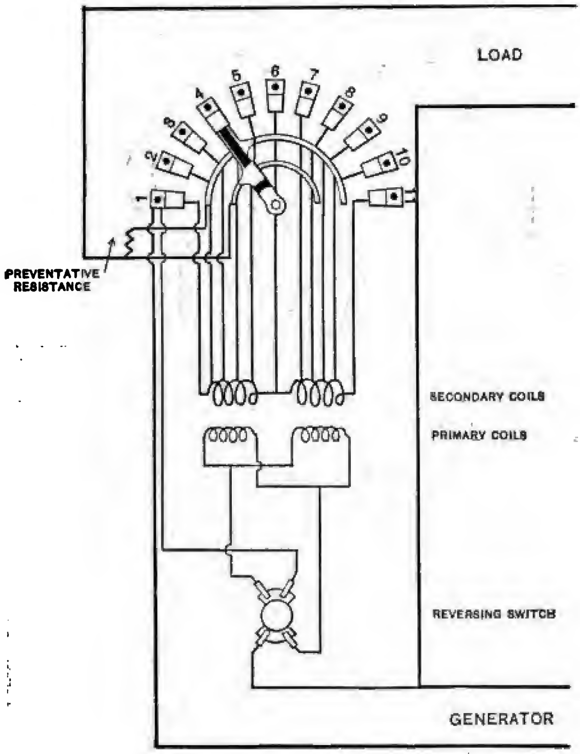

FIG. 46*D*. Connections of a Stillwell regulator.

ing the direction in which the flux threads the secondary. If this is accomplished with fixed windings and a movable or partly movable magnetic circuit, the apparatus is called a magneto-potential regulator; if by a relative motion of primary and secondary circuits, it is termed an induction pressure regulator.

Figure 46*E* illustrates a regulator of the magneto type. The primary and secondary coils are disposed in planes which intersect at right angles on a common axis of the two coils. If, when the inductor is in the position shown, the secondary pressure is additive, it will be of opposite sign for a position at right angles

to this. The secondary pressure is a maximum for the 45 degree positions of the inductor, zero for either the vertical or

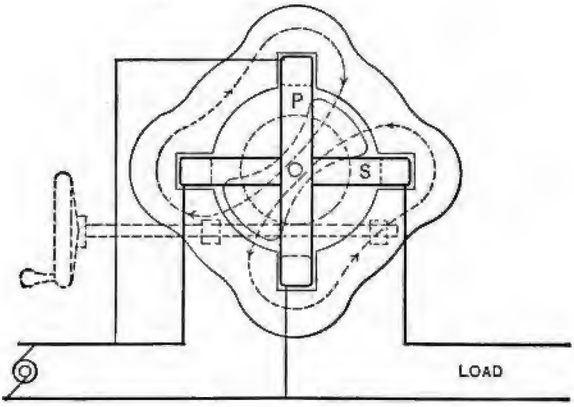

FIG. 46*E*. Magneto-potential regulator.

the horizontal positions, and of an intermediate value for any other angle. The induction pressure regulator is well illustrated

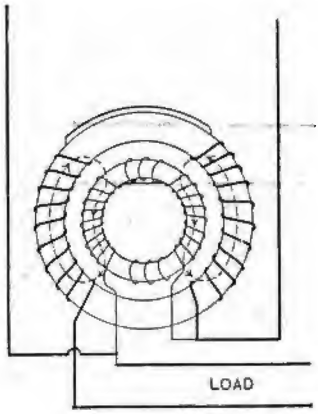

FIG. 46*F*. Induction pressure regulator, raising potential.

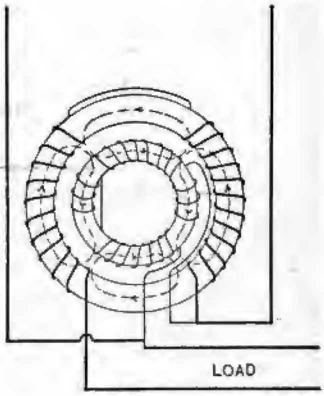

FIG. 46*G*. Induction pressure regulator, neutral position.

in Figures 46*F*, 46*G*, 46*H* and 46*I*. In this type, the relative position of the primary and secondary coils may be changed by an external device controlled by the operator. In this par-

ticular case the primary is movable and the secondary is stationary, the construction representing that of an induction motor. The movable core has wound upon it, at diametrically opposite positions, several closed circuited turns designed to prevent a magnetic short circuit when the regulator is at zero voltage, or what is termed the neutral position. This magnetic short circuit is readily seen from the fact that in the neutral position the primary and the secondary currents conspire to produce a flux in

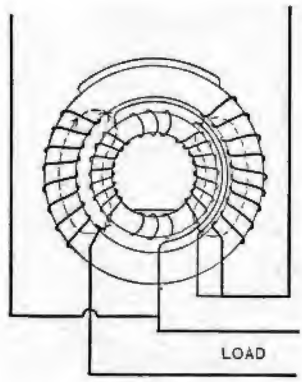

FIG. 46*H*. Induction pressure regulator, lowering potential.

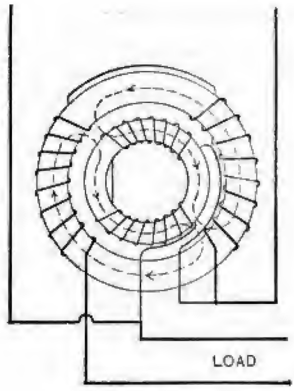

FIG. 46*I*. Induction pressure regulator, short circuiting coil removed.

the same direction, thus causing a transformation. This is illustrated in Figure 46*I*, where the short-circuiting coil is removed.

Data. With the primary voltage normal and constant, and a non-inductive secondary load, make the necessary observations for efficiency and regulation of a transformer pressure regulator, using both the intake and output method and the stray power method. Make tests covering every condition under which the regulator is likely to be used in practice.

No. 47. THE REACTANCE COIL AS A POTENTIAL REGULATOR.

References. Bedell, p. 324; Fleming's "Transformers," Vol. 2, pp. 190 to 193; Arnold, Vol. 2, pp. 126 and 127.

Object. To test some forms of reactance coil.

Theory and Method. The reactance coil in its elementary form is shown in Figure 47*A*. It is here connected in series with the load. In this case it is well to take the current as the standard of phase reference.

It will require a definite pressure E to send a current I through this coil. The component of pressure in phase with the current is the active pressure; for, when multiplied by I, the product will be the I^2R loss of the coil plus the core losses. This current I produces a certain number of ampere turns and the resultant intensity of magnetization will depend on the reluctance of the magnetic circuit. This magnetization produces an electromotive force of self-induction lagging behind it by 90 degrees, and the pressure required to overcome this pressure leads by 90 degrees. The resultant of this component and the equivalent IR drop is the pressure E.

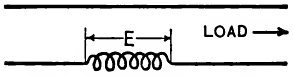

FIG. 47*A*. Reactance coil in series with a line.

It is thus seen that *for a given current*, the higher the reluctance of the magnetic circuit, the less the reactance and the smaller the pressure absorbed; while, *for a given pressure*, the higher the reluctance the lower the reactance and the greater the current required to produce the necessary flux.

The wattless component of pressure will be in direct proportion to the frequency as long as the permeability remains constant and it would, therefore, take about half the turns to produce the same choking effect at 120 cycles as at 60 cycles.

These reactance coils are generally placed in series with a load, the function being that of a pressure regulator. They are used in series with feeders for lighting and power systems; as theatre dimmers, "economy coils" in alternating arc lamps, and in similar ways.

The reactance coil is plainly a phase-displacing device. The American Institute of Electrical Engineers has defined the efficiency of phase-displacing apparatus as the ratio of the volt-

ampere intake minus the losses, to the volt-ampere intake. This is plainly an extension of the technical meaning of "efficiency" and should not be confounded with the term as applied to the transformation of energy.

The losses in impedance coils vary as some function of the current, frequency remaining constant.

Any test should be made with connections for maximum reactive effect and with a non-inductive load, unless otherwise specified.

Figure 47*B* shows a variable reactance produced by commutating the turns of the coil. The choke coil of an alternating cur-

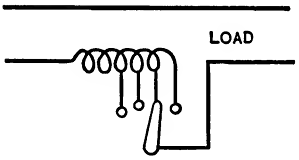

Fig. 47*B*. Variable reactance in series with a line.

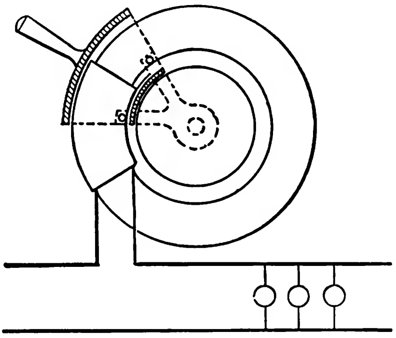

Fig. 47*C*. Thomson potential regulator.

rent constant pressure arc lamp, with adjustment for frequency and pressure, is an application of this arrangement.

Figure 47*C* represents a variable reactance, the design of which is due to Professor Elihu Thomson. It has been used as a voltage regulator on series incandescent systems. It consists of a transformer whose primary is in series with the load, and whose secondary is movable and consists of a single short circuited turn. When the secondary is swung so as to completely enclose the primary, the mutual inductance is a maximum and the reactive effect a minimum. When moved to a position diametrically opposite, the reactive effect is a maximum. This

coil gives a smooth gradation of regulation, but the advantage is largely offset by the excessive secondary loss. Figure 47*D*

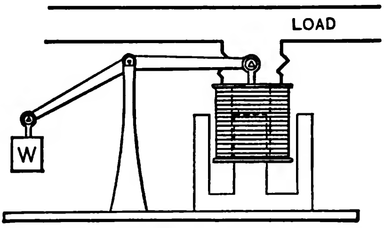

FIG. 47*D*. Series arc light regulator.

represents a form in which the amount of reactance is changed by varying both the reluctance and the leakage of the magnetic circuit. The reactance is a maximum when the coil is down, and a minimum when the coil is up. This device is used as a regulator for series arc lighting.

Data. Take observations for a regulation curve of an impedance coil, set for its maximum reactance, and using a non-inductive load. Separate the losses as far as possible, and calculate the efficiency on the basis of volt-ampere activity.

Curves. Plot curves of efficiency and regulation.

No. 48. STUDY AND TEST OF AN AUTO-TRANSFORMER OR COMPENSATOR.

References. Bedell, pp. 324 to 330; Karapetoff, p. 426; Standard Handbook, Sec. 6, Art. 133 to 141; Thompson's "Dynamos," Vol. 2, pp. 603 and 604; Esty, pp. 220 to 222; Arnold, Vol. 2, p. 124; Russell, Vol. 2, pp. 289 to 292; Lamb, Chap. 6.

Object. The auto-transformer being used in many special applications where a transformer could be employed, it is well to compare it with the latter.

Theory and Method. Figure 48*A* represents a step-down transformer and Figure 48*B* shows an auto-transformer intended for the same service. The chief difference is that in the latter

the secondary winding forms a part of the primary. It is obvious that in house to house service the auto-transformer is objectionable because a ground on one of the high pressure mains

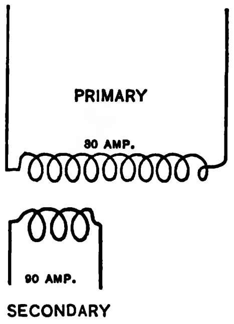

FIG. 48*A*. Transformer for lowering voltage by a 3 to 1 ratio.

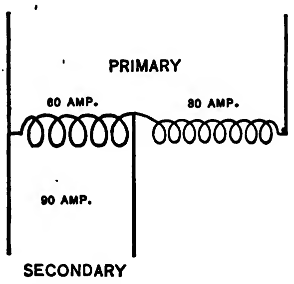

FIG. 48*B*. Auto transformer for lowering the voltage by a 3 to 1 ratio.

would cause a strain on the insulation of the secondary distribution and would be a menace to safety. While a system of connections like that of Figure 48*C* would minimize this strain, it would even then be prohibitive. Referring to Figure 48*A*, and neglecting magnetizing currents and losses, assume a ratio of three to one and a load of 90 amperes. For the same current densities the primary and secondary cross sections of conductor would be represented by 1 and 3, respectively, and the lengths by, say, 300 and 100; the same length of mean turn being presupposed.

In the transformer, the primary and the secondary currents are practically at a phase difference of 180 degrees on non-inductive loads. This is also true of the auto-transformer. Therefore, it is evident that, in Figure 48*B*, the current in the secondary coil will be the difference between the primary and secondary currents, in this case 60 amperes, and the primary current will be 30 amperes as before. The primary and secondary cross sections will then be represented by 1 and 2 respectively, and the lengths by 200 and 100. Comparing the quantities of copper, we now have:—

Transformer .... $300 \times 1 + 100 \times 3 = 600$,
Auto-transformer .... $200 \times 1 + 100 \times 2 = 400$,

or a saving of 33 percent. in favor of the auto-transformer. The total cross section of copper being reduced, the mean turn will be shorter and a still further saving in copper will result. The

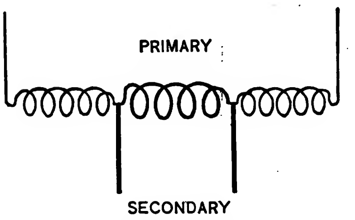

FIG. 48*C*. Auto transformer to limit the strain in the low potential system.

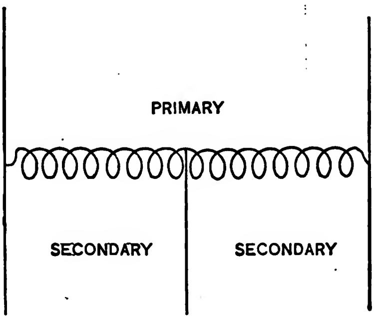

FIG. 48*D*. Auto transformer for a three wire system.

resistance being smaller the copper loss will be less, or, designing for the same copper loss, a still lighter construction is permissible. Furthermore, this means a saving in the volume of iron, as the winding space may be made smaller. The volume of iron being less, the core losses will be less for the same density and the auto-transformer will have a higher efficienciy; or a higher density may be used, thus keeping the efficiency the same, and a still further saving in iron and copper be effected.

It is thus seen that the auto-transformer can be made lighter and with a less expensive construction than an ordinary transformer of the same capacity. The saving in weight depends upon the ratio of transformation, the limit being one hundred percent. when the ratio is unity. The question of temperature would demand a certain amount of radiating surface, however, so that the compactness of the apparatus might not be so great as this discussion would indicate; but this, together with the character of service (intermittent or continuous), are matters of special design rather than of inherent working properties.

For good regulation the primary and secondary coils should be wound in sections and interleaved the same as in transformers.

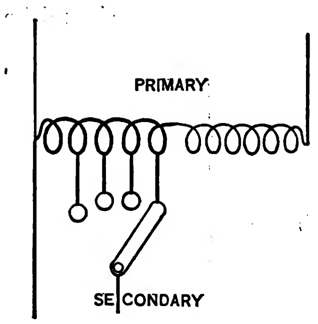

FIG. 48*E*. Auto transformer as a voltage regulator or a motor starter.

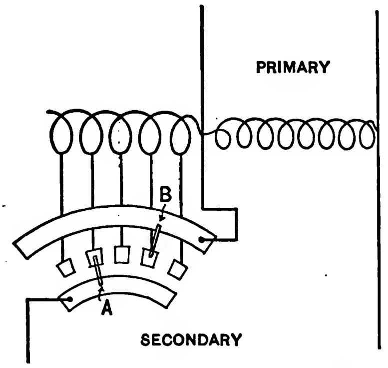

FIG. 48*F*. Auto transformer as a voltage regulator for raising or lowering the feeder voltage.

From what has been said it is evident that the field of usefulness for the auto-transformer is where the ratio of transformation is small, all wiring being then put in for the same fire risk. It is thus we find the auto-transformer used as a potential regulator, a starting device for alternating current motors, or a compensator on the multiple wire system.

The commercial forms of the auto-transformer are legion. Figure 48*D* represents a compensator for a three-wire system. Its use in this service is now a matter of ancient history, primary voltages having been raised so high as to crowd it out of the commercial field. Figure 48*E* represents a voltage regulator, or a motor starter, and Figure 48*F* still another form. The contacts *A* and *B* are geared together to move equal amounts in opposite directions. Figure 48*G* is a form having the same range of transformation as Figure 48*F*, but twice the number of secondary turns.

Probably the most accurate method of testing the efficiency of an auto-transformer is, first to make the regulation test, and second, from the observed values of primary and secondary cur-

rents, to calculate the copper losses from measured resistances, care being taken that the resistances are measured between the proper terminals.

The core loss should be measured on open circuit. If the primary has a variable number of turns, this core loss should be

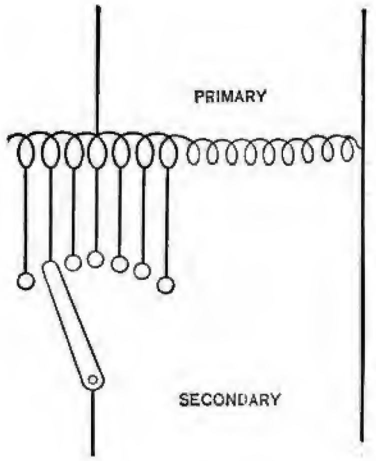

FIG. 48*G*. Auto transformer for raising or lowering the feeder voltage.

measured at the maximum ratio of transformation and, unless otherwise specified, the entire test should be made for the maximum voltage of the variable coil and on a non-inductive load. Temperature tests are conducted on the same basis as in the transformer test.

Suggestion. Since auto-transformers are usually operated on inductive loads, this feature should be kept in mind in testing one for a particular service.

Data. Obtain data for regulation and efficiency at various loads by the stray power method, operation being at maximum voltage on the variable coil and on a non-inductive load.

Curves. Plot curves of regulation and efficiency, using percent. load as abscissas.

Question. Would you expect to find compensators in general

designed with the same care as to regulation as are transformers? Give reasons.

INTRODUCTION TO EXPERIMENTS ON SYNCHRONOUS MACHINES.

The electromotive force generated in the armature of an alternator depends upon the rate of change of flux through the armature coils. At no load, its maximum value occurs when the conductors are under the centers of the poles. The value of the flux from a field pole depends upon the magnetomotive force and the reluctance of the magnetic circuit.

When current flows in the coils of the armature, there is produced a fall of potential proportional to the current and in phase with it, due to the resistance of the coils, and also a fall of potential at right angles with the current, due to the reactance of the coils. The reactance is due to the inductance of the armature coils caused by leakage lines around the imbedded conductors and also around the end conductors. The inductance is nearly independent of the value of the armature current but depends upon the position of the coil. It is usually greater when the conductor is under the center of a pole face. The resistance drop is usually small in comparison with the reactance drop.

In Figure S1 let

E' = generated emf.,
E = terminal emf.,
I = current in the armature,
R = armature resistance,
X = armature reactance $= 2\pi f L$,
Z = armature impedance,

and let the load be non-inductive. The current lags behind the electromotive force E' by the angle β', Figure S1, and that E' is greater than E by the value of impedance drop IZ taken in its proper phase relation with respect to the current. Figure S2

shows a developed section through the armature and poles. As stated before, the electromotive force will reach its maximum in the position shown by the circles representing the conductors in

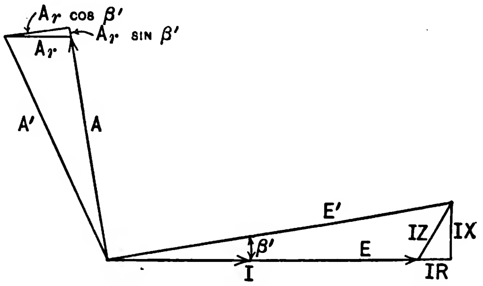

FIG. S1. Electromotive force and magneto-motive force diagram of an alternator, non-inductive load.

the slots. It will be induced in the direction indicated in the circles. The current will reach its maximum somewhat later, after the conductors have passed over an angle of β' electrical degrees.

There will result two effects produced by the armature reaction

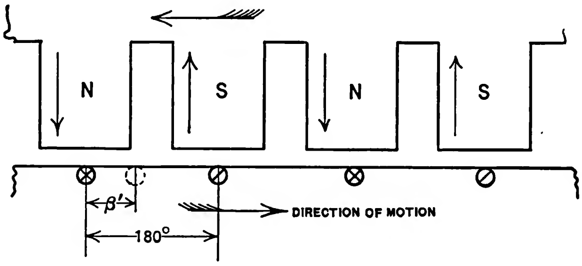

FIG. S2. Section through developed field and armature of a generator, non-inductive load.

or the magnetomotive force of the armature conductors. There will be a distorting action due to the component of the current $I \cos \beta'$ in phase with the induced electromotive force, and a demagnetizing action due to the component $I \sin \beta'$. Referring to Figure S1, this may be shown vectorially in the following manner. Let

A = the field ampere turns necessary to produce E' on open circuit, and

A_r = the armature ampere turns when the current is I flowing. The field ampere turns will be in phase with the flux and hence at right angle with the induced electromotive force E'. A_r will be in phase with the armature current. These two magnetomotive forces acting on the same magnetic circuit, produce A', the ampere turns in the field necessary to produce the electromotive force E with the current I flowing, the load being non-inductive. It will also be seen that

$$A' = \sqrt{(A + A_r \sin \beta')^2 + (A_r \cos \beta')^2}.$$

$A_r \sin \beta'$ is the demagnetizing component of the armature reaction. $A_r \cos \beta'$ produces a distorting action on the field. This tends to distort the flux causing the leading pole tip to be the stronger. This in turn distorts the electromotive force wave.

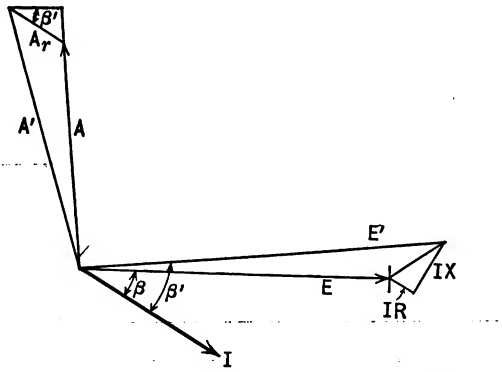

FIG. S3. Electromotive force and magneto-motive force diagram of an alternator, inductive load.

Figure S3 shows a similar construction for an inductive load. It will be seen that the impedance drop comes more nearly into phase with the terminal electromotive force, the greater the lag angle. Hence, the greater the angle of lag, the greater the difference between the generated and terminal electromotive forces,

for the same armature current. The armature reaction causes a greater demagnetizing and a less distorting action, the greater the angle of lag. This is true because the axes of the armature

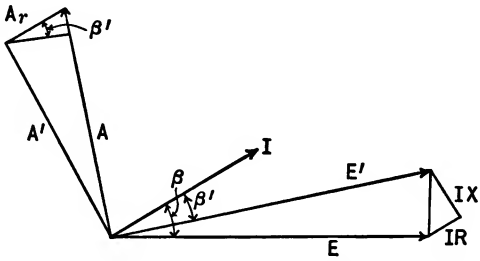

FIG. S4. Electromotive force and magneto-motive force diagram of an alternator, capacity load.

coils come more nearly into line with the axes of the field poles when the armature current reaches its maximum.

Figure S4 shows the effect of capacity load on an alternator. The capacity load not only acts to conterbalance the effect of

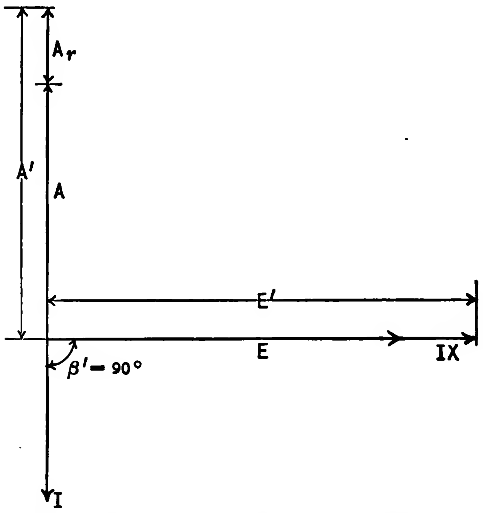

FIG. S5. Electromotive force and magneto-motive force diagram of an alternator, current lagging 90°.

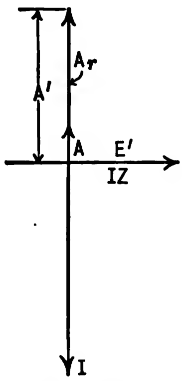

FIG. S6. Electromotive force and magneto-motive force diagram of an alternator on short circuit.

the inductance in the armature, but also causes a compounding effect on the fields. This latter is true, since $A_r \sin \beta'$ is added to the field ampere turns when the capacity is more than sufficient to balance the armature inductance.

A peculiar condition exists when the current lags 90 degrees behind the terminal electromotive force as in Figure S5. The resistance drop is small in comparison with the reactance drop and the impedance drop may be added directly to the terminal electromotive force to give the induced electromotive force. The ampere turns of armature reaction are subtracted from the total field ampere turns to give the value of the ampere turns necessary to produce E' on open circuit.

An alternator on short circuit corresponds to this condition,

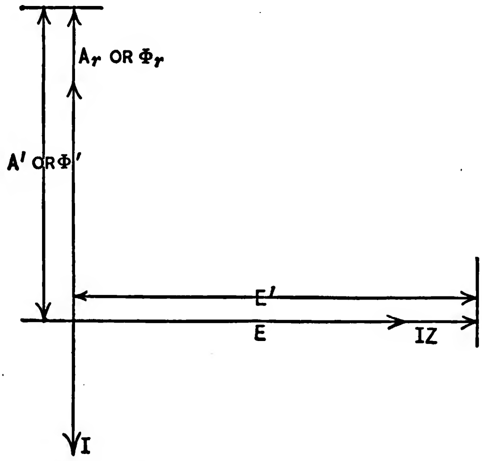

FIG. S7. Electromotive force, magneto-motive force and flux diagram of an alternator having a constant magnetic reluctance.

since, as the resistance drop is small, the current must lag nearly 90 degrees behind the induced electromotive force. In this case the terminal electromotive force E is zero, and the only electromotive force existing in the circuit is the value of the impedance drop IZ, Figure S6. The armature ampere turns A_r will be subtracted directly from the total field ampere turns A', to give the value of A. A' is then the value of the field ampere

turns necessary to send the current I through the armature, when on short circuit. If the value of the electromotive force be found, corresponding to A', Figure S5, with the armature on open circuit, as E'', then

$$\frac{E'' - E}{I} = Z',$$

a new value of impedance. If the reluctance of the magnetic circuit were constant, this value could also be derived by finding the electromotive force on open circuit corresponding to A' in Figure S6. The value of Z' found in this latter manner is known as the *synchronous impedance* of the alternator. The synchronous impedance may be found in the following way. The field current is found which is necessary to produce full load current, at normal frequency, with the armature on short circuit. The armature is then placed on open circuit and the value of the electromotive force found at normal frequency and the same field current. This electromotive force, divided by the full load armature current, gives the synchronous impedance. This value of syn-

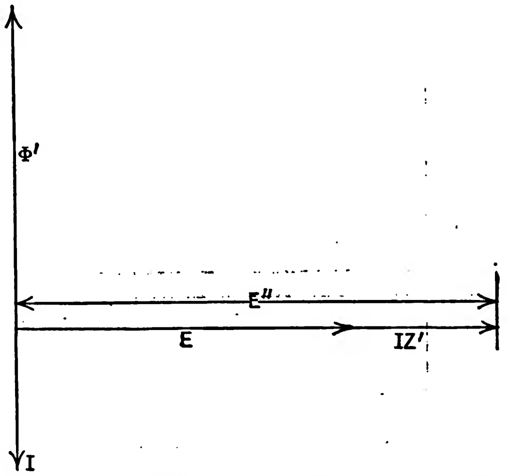

FIG. S8. Electromotive force and flux diagram of an alternator having a constant magnetic reluctance. Synchronous impedance drop in proper phase relation.

chronous impedance is nearly constant for all loads. It is useful for the purpose of making a predetermination of the behavior of the machine run as an alternator or as a synchronous motor.

If the magnetic circuit has a constant reluctance, then flux may be substituted for ampere turns in each case, and Figure S5 becomes Figure S7. If, now, we consider the demagnetizing effect of the armature current in the equation for the synchronous impedance, we have Figure S8. In this diagram we have a new value of flux ϕ', a new value of electromotive force E'' and the synchronous impedance drop ZI. ϕ' is the flux which would be

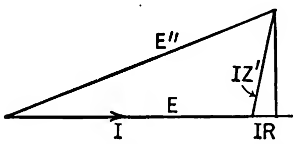

Fig. S9. Electromotive force diagram of an alternator, non-inductive load.

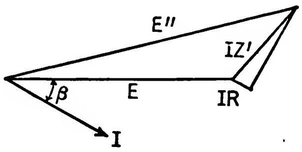

Fig. S10. Electromotive force diagram of an alternator, inductive load.

produced by the ampere turns A' with the armature on open circuit. E'' is the electromotive force produced at normal frequency by the flux ϕ', with the armature on open circuit. No such values of flux, or electromotive force, exist, but a fairly close approximation to the value of terminal voltage E and the voltage regulation may be made by using them. Figures S9, S10 and S11 show the construction for non-inductive, inductive and capacity loads, using the fall of potential due to synchronous impedance in its proper phase relation with the armature current.

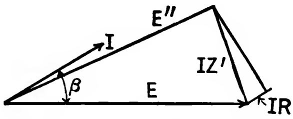

Fig. S11. Electromotive force diagram of an alternator, capacity load.

It will be seen, by studying these diagrams, that the variation of the voltage at the terminals of an alternator as the load changes, the field current remaining constant, depends not only upon the constants of the machine but also upon the character of the load. This change in voltage from full load to no load in terms of the full load voltage is known as the regulation of the alternator.

The Synchronous Motor.

If two similar alternators be run at the same frequency and have the same terminal electromotive force, they may be made to operate in parallel and the load will be properly divided between them. The process of putting the machines in parallel is known as synchronizing. The methods of synchronizing will be considered in the experiments relating to that subject.

If the driving force be removed from one of the alternators when it is operating in parallel with other machines, it will continue to run as a motor at constant speed and may be made to supply mechanical power from its shaft. This type of motor has the property of running at absolutely constant speed, and of furnishing inductive, or capacity loads, to the system according to the excitation of its fields.

To understand this property of the synchronous motor, let us consider the following diagrams.* The quantity known as the synchronous impedance has just been defined. In Figure S12 let OI be taken as the direction of the current vector. Let

$E =$ the terminal electromotive force,
$E' =$ the generated electromotive force,
$Z' =$ the synchronous impedance,
$R =$ the armature resistance,
$I =$ the armature current,
$\beta_1 =$ the angle between current and terminal electromotive force,
$\beta_2 =$ the angle between current and generated electromotive force.

Then $\cos \beta_1 =$ the power factor of the motor.

Assume the terminal electromotive force to be constant. This may be represented by the locus, a circle about O with E as a radius. The impedance drop IZ' and the resistance drop IR

* The mathematical development is not attempted here, as the mathematics are complete in the references given in the experiments bearing upon the subject.

will have a constant ratio to each other independent of the load; that is, the angle θ between IZ' and the current will be constant.

Consider the point B on the diagram. The current is in phase with the terminal electromotive force. There will be formed

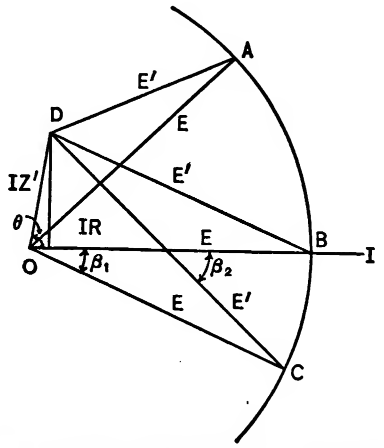

Fig. S12. Electromotive force diagram of a synchronous motor.

a triangle between E, E' and IZ', the triangle BOD. The length OD will depend upon the value of the armature current. The position of point B and the angular position of E' will depend upon the motor excitation.

If, now, the motor excitation be decreased and the current be kept constant, the point B must move along the circle in the direction of A. It will be seen that this produces a lagging current at the terminals of the machine. If the excitation of the machine had been increased, the point would have moved to C, giving a leading current.

The intake, represented by $EI \cos \beta_1$, must be decreased with either decreasing or increasing field excitation, the current remaining constant. Or better, if the load is kept constant, the armature current must increase for any departure in the field current from that value making E and I in phase. This increase is due to the wattless component in the armature conductors necessary to produce the proper armature reactions, as in the case of the same

machine operated as an alternator. Here it will be seen that the lagging current is magnetizing and the leading current is demagnetizing. It must also be remembered that the current reverses with respect to the terminal electromotive force in changing from a generator to a motor.

A more convenient diagram for studying the behavior of a synchronous motor is that of Professor Blondel. He makes use of a form of circle diagram shown in Figure S13. The triangle AOD is similar to that in Figure S12. On OA erect an isosceles triangle having base angles θ. The vertex C of this triangle is

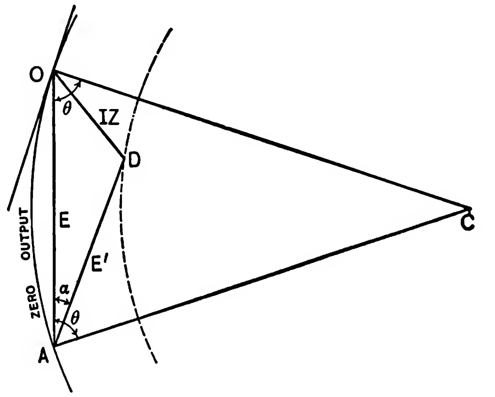

Fig. S13. Circle diagram for a synchronous motor.

the center of a circle passing through O and A. Concentric circles about C represent the output of the motor, including the core and friction losses. In the consideration of this diagram the values of θ, Z' and E are assumed to be constant. The length AD or E' is the generated electromotive force. The angle between OD and OC represents the angle of lag or lead of the motor current. The position of D on OC represents unity power factor. Any position of D below OC represents lagging current and above OC leading current.

As the load on the motor is increased and the excitation kept constant, the length OD must increase and D moves on the arc of a circle about A as a center. Motor intake is represented by

the straight lines perpendicular to OC and the motor output by circles about C, the circle through O representing zero output. Circles about O represent armature current with a value equal to OD divided by Z'. Any position of D beyond a diameter through A and C represents unstable operation. By the aid of this form of diagram many of the operating features of a synchronous motor may be studied.

An adaptation of this type of diagram is made by Professor C. A. Adams. He uses corresponding magnetomotive forces, or field currents, instead of electromotive forces. Making these assumptions in Figure S13, OA is the field current corresponding to the terminal voltage on open circuit, OD the field current corresponding to a given value of armature current, the armature being on short circuit, and AD the field current corresponding to a given excitation of the motor under load. The isosceles triangle AOC is constructed as before with θ for its base angles. This diagram brings more accurate results than the electromotive force diagram but is slightly harder to interpret.

No. 49. OPEN CIRCUIT SATURATION CURVE OF AN ALTERNATOR.

References. Sever and Townsend, pp. 125 and 126; Thompson's "Dynamos," Vol. 2, p. 275; Esty, pp. 155 to 158; Russell, Vol. 2, pp. 23 to 28; Thomälen, p. 203; Franklin and Esty, p. 132; Arnold, Vol. 4, pp. 631 to 632 and 3 to 16.

Object. To obtain the necessary data for the open circuit saturation curve of an alternator at normal frequency. To determine the effect of variation of speed of the alternator upon the shape of the curve.

Theory and Method. The electromotive force at the terminals of an alternator on open circuit depends upon the fundamental equation

$$E = \frac{\sqrt{2}\,\pi f n \phi k}{10^8}, \tag{49a}$$

where

f = frequency,

n = number of turns in series.

ϕ = flux per pole, and

k = a constant depending upon the grouping of the turns.

The frequency of the alternator is defined as the number of like poles passed by a given point on the armature in one second, or

$$f=\frac{Vp_1}{60}, \tag{49b}$$

where

V = revolutions per minute, and

p_1 = number of like poles in the field.

From Equation 49*a* it is seen that, for a given grouping of the coils on the armature and a given frequency, the terminal electomotive force varies directly with the flux ϕ. A curve between the open circuit electromotive force and the ampere turns on the field (sometimes field current), at constant frequency, is called the open circuit saturation curve. It may also be seen that, with constant flux ϕ, the open circuit electromotive force varies directly with the frequency. The saturation curves of the same machine at different frequencies will have different shapes if plotted to the same scale.

Data. Drive the alternator at constant speed and take readings of terminal electromotive force and field current for increasing values of field current from zero to a value somewhat above normal (25 percent. or more), then for decreasing values of field current back to zero. Secure the winding data of the machine, if possible, in order to check your values.

Curves. Plot an open circuit saturation curve of the machine, using electromotive force as ordinates and field ampere turns, or field current, as abscissas. Plot a similar curve for the machine at double frequency.

Questions. How is the effect of hysteresis shown by your curve? Does this curve differ from a similar curve taken for a

transformer? Why? In what part of the magnetic circuit does the hysteresis loss causing heating occur? Is the area of this magnetic loop a measure of this loss?

No. 50. SYNCHRONOUS IMPEDANCE OF AN ALTERNATOR ARMATURE.

References. Steinmetz' "A.C. Phenomena," Chap. 22; Steinmetz' "Elements," p. 136; Russell, Vol. 2, pp. 54 and 55; Sever and Townsend, pp. 122 to 124 and 130; Esty, p. 158; Thompson's "Dynamos," Vol. 2, p. 275; Thomälen, pp. 298 to 304; Berg, pp. 89 to 91; Franklin and Esty, p. 133; *Elec. Wld. and Eng.*, April 26, 1902, F. G. Baum, Synchronous reactance.

Object. To determine the synchronous impedance of an alternator armature.

Theory and Method. The armature reaction and armature reactance may be grouped together in one term known as the *synchronous reactance*. The synchronous reactance is a fictitious quantity useful in graphical construction and mathematical analyses of the operation of a synchronous machine. It is usually considered as constant over the working range of load on any machine.

In the determination of synchronous reactance, the alternator is operated at its normal frequency and the armature terminals are short circuited through an ammeter of low impedance. The fields are slightly excited and the field current increased until the required armature current I flows. The field and armature currents are then read. With the alternator operating at the same frequency and the field current maintained at the value just determined, the armature circuit is opened and the voltage E' is read on a voltmeter. This value of electromotive force may also be obtained from the open circuit saturation curve of the machine at the same frequency. The synchronous impedance is then

$$Z' = \frac{E'}{I}.$$

Since the armature resistance may be neglected in comparison with the reactance, this may also be written

$$X' = \frac{E'}{I}.$$

The synchronous reactance of many alternators is of such a value that about three times full load current will flow on short circuit with the fields normally excited. In the case of inductor alternators it frequently occurs that the current is less than full load under the same condition. More than three times full load current will flow in the case of turbo-alternators.

Caution. Do not excite the field to a point which will cause the armature current to exceed a safe value. Use an ammeter of low impedance compared with that of the armature, and connect by means of low resistance wires. If the armature circuit is opened, while the fields are excited, do not close the circuit again without first lowering the excitation of the machine, or the armature and instruments may be damaged by the momentary rush of current.

Data. Running the machine at a frequency near normal, adjust the field current for about 50 percent. overload current in the armature. Take readings of armature current and field current from 50 percent. overload to no load, varying the readings by convenient steps. If the saturation curve at the rated frequency has not been obtained, it should be taken as explained in Experiment 49. Measure the armature resistance.

Curves. Plot the following curves with field current as abscissas:

Synchronous reactance and field current,
Armature current and field current,
Open circuit voltage and field current.

Also plot the curve between open circuit voltage and armature current for the same values of field current, using armature current as abscissas.

Questions. How would you make the necessary corrections, if the resistance of the ammeter and leads is appreciable? If the field current be left constant and the alternator gradually slowed down, the armature current will also remain almost constant, until the machine has come nearly to a standstill. Explain why this is so. What is the cause of the momentary rush of current mentioned above?

Note. If the alternator is polyphase, it is well to compare the ratio of armature reactions when connected single phase and polyphase. This may be done by determining the value of the synchronous reactance in each case for the same value of armature current.

No. 51. REGULATION OF AN ALTERNATOR UNDER VARIOUS CONDITIONS OF LOADING.

References. Karapetoff, pp. 485 to 488; Russell. Vol. 2, pp. 59 to 61; Thompson's "Dynamos," Vol. 2, pp. 265 and 276 to 278; Steinmetz' "A.C. Phenomena," Chap. 21 and 22; Steinmetz' "Elements," pp. 125 to 141; Esty, pp. 86 and 164; *Tech.*, No. 15, 1900 and 1901, G. W. Redfield, Regulation of alternators under reactive loads; Arnold, Vol. 4, pp. 347 to 350 and 636, also Chap. 3; *Am. Elect'n,* August, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.,* October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.,* May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; April 22, 1904, H. M. Hobart and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.,* December 17, 1904, C. F. Guilbert, On the theory of the regulation of alternators; *Trans. Int. Elec. Cong.,* St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rush-

more, The regulation of alternators; *Elec. Wld. and Eng.*, September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.*, October 31 and November 28, 1903, B. A. Behrend, Testing of alternating current generators.

Object. To obtain the external characteristic of an alternator under various conditions of loading.

Theory and Method. As explained in the Introduction to Experiments on Synchronous Machines, the character of the load in the external circuit of an alternator, as well as the constants of the machine, determines the shape of the external characteristic under load. The change in pressure at the terminals as the armature current is increased, depends upon the armature resistance, armature inductance and armature reaction.

When the current in the armature is in phase with the electromotive force, the field flux is distorted and suffers but slight change in the total value. When the current lags behind the electromotive force, the current in the armature coils produces a magnetomotive force opposing that of the fields. It therefore follows that an inductive load on an alternator causes the external characteristic to drop more rapidly than in the case of a non-inductive load.

If the current leads the electromotive force, the armature reaction tends to strengthen the field magnetization of the alternator. In consequence, a capacity load causes a compounding effect. This tends to increase the terminal pressure as the load increases.

Suggestions. A power factor of 80 percent. represents about the limit of good practice. The power factor may be computed, or measured by means of a power factor indicator. In the case of an inductive load it may be maintained constant by manipulating a variable inductance in parallel with a variable resistance. If the speed changes with the load, it is quicker to adjust the power factor first and then the speed. In the case of a capacity

load condensers in parallel with a variable resistance may be used. If sufficient capacity at the rated voltage of the machine is not available, use may be made of a transformer to give the proper voltage on the condensers. It must be remembered that the capacity current of the condensers will be multiplied by the square of the ratio of transformation.

A synchronous motor may be used to produce the inductive and capacity loads on the alternator. In this case the shape of the current wave will not be as much affected as with a load consisting of condensers or inductances. The motor should be run without load and the load supplied by means of a variable resistance in parallel with it.

It is well to make the alternator used in this experiment your standard machine for all regulation experiments. This will enable comparisons of the various methods of determining regulation to be made and will save the duplication of a number of observations.

Data. Take readings for loads varying from 50 percent. overload current to no load, first with a non-inductive load and then with inductive and capacity loads of 80 percent. power factor. Adjust the machine each time for normal voltage at full load current and rated speed. Take readings of terminal pressure, load current, field current and speed from 50 percent. overload current to no load, maintaining the speed, field current and power factor constant for each adjustment at full load current.

Compute. The percent. regulation from the full load and no load observations for each of the different sets of readings.

Curves. Plot a curve, for each condition of loading, between terminal pressure and load current, using load current as abscissas.

No. 52. EXCITATION CHARACTERISTIC OF AN ALTERNATOR.

References. Karapetoff, p. 489; Thomälen, pp. 305 to 306; Standard Handbook, Sec. 7, Art. 82; *Am. Elect'n,* August, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.,* October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.,* May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; April 22, 1904, H. M. Hobart and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.,* December 17, 1904, C. F. Guilbert, On the theory of the regulation of alternators; *Trans. Int. Elec. Cong.,* St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rushmore, The regulation of alternators; *Elec. Wld. and Eng.,* September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.,* St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.,* October 31 and November 28, 1903, B. A. Behrend, Testing of alternating current generators.

Object. To show the increase in field ampere turns necessary to maintain the terminal pressure constant for various loads.

Theory and Method. One of the problems which the engineer meets in connection with alternating current generators is to keep the terminal pressure constant under all conditions of loading. This is accomplished by adjustment of the field excitation to compensate for the effect of armature reaction and inductive drop. The excitation characteristic shows the value of field ampere turns necessary to keep the terminal voltage constant with different loads upon the machine. As explained in the Introduction to Experiments on Synchronous Machines, the character of the load, as well as the amount of the armature current, determines the field ampere turns necessary for a given voltage. The relative saturation of the magnetic circuit also determines the effect of the armature reaction on the terminal voltage.

Data. Adjust the field excitation of the machine for normal voltage at 50 percent. overload current with a non-inductive load and at the rated frequency. Take readings of armature current and field current from this load to no load, maintaining the speed and terminal pressure constant. Measure the resistance of the fields when hot. Repeat, using a value of terminal voltage less than normal; also with inductive and capacity loads of 80 percent. power factor.

Curves. Plot the excitation characteristics, taking field ampere turns as ordinates and armature current as abscissas, for the different conditions of loading.

Suggestions. An excitation characteristic might be taken for other conditions than those assumed in this experiment, such as a given rise in pressure with load, either with constant power factor or with some predetermined gradient of the power factor. These conditions, while complicated, are often encountered in practice.

Question. What would be the rating of the field rheostat of the alternator, for non-inductive load, given in resistance and current carrying capacity?

Compute. The proper resistance steps to be used in such a rheostat for the condition of normal voltage non-inductive load. Assume 60 steps in the rheostat.

No. 53. FULL LOAD SATURATION CURVE OF AN ALTERNATOR.

References. Sever and Townsend, p. 125; Russell, Vol. 2, pp. 55 to 59; Thompson's "Dynamos," Vol. 2, pp. 275 to 280; Standard Handbook, Sec. 7, Art. 81; Arnold, Vol. 4, pp. 347 and 633; *Am. Elect'n,* August, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.,* October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.,* May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regu-

lation of alternators; April 22, 1904, H. M. Hobart and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.*, December 17, 1904, C. F. Guilbert, On the theory of the regulation of alternators; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rushmore, The regulation of alternators; *Elec. Wld. and Eng.*, September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.*, October 31 and November 28, 1903, B. A. Behrend, Testing of alternating current generators.

Object. To plot the load saturation curve of an alternator.

Theory and Method. As explained in the Introduction to Experiments on Synchronous Machines, the effect of the armature reaction in an alternator is to distort the field flux or to oppose this flux, depending upon the phase of the armature current with respect to the generated electromotive force. The effect of the resistance of the armature upon the terminal voltage also depends upon the same condition. The degree of saturation of the magnetic circuit is another determining factor in the effect of the armature reaction on the terminal voltage of the machine. It is the purpose of this experiment to plot the curve between terminal voltage and field current for a constant value of armature current and power factor. Such a curve is known as the load saturation curve of the machine. Obviously there is an infinite number of such curves, depending upon the conditions chosen. In order to have these curves available for future calculations, certain ones will be chosen here. The conditions to be considered in a given problem will aid in determining the curves to be used.

One curve to be chosen is that for inductive load with full load current and zero power factor. This curve will be useful in determining the regulation by methods considered later. The same effect will be produced with a load of 20 percent. power factor, or less, as with a load of zero power factor. This power

factor may be secured by means of either an unloaded induction motor or a synchronous motor underexcited.

Data. Load the alternator with a synchronous motor. Starting with normal frequency and a value of terminal voltage somewhat above normal, and with the synchronous motor underexcited so as to produce full load current and a power factor of 20 percent., or less, take readings of terminal voltage and field current, keeping the frequency, armature current and power factor constant. The field current of the alternator should be reduced by steps to as low a value as the synchronous motor will operate, and readings should be taken at frequent intervals. Finally, determine the value of field current necessary to produce full load current with the armature short circuited. Repeat, using full load current and a non-inductive load. Take readings for an open circuit saturation curve as explained in Experiment 49. These readings should be taken very carefully, as they form the basis of several later experiments.

Curves. Plot curves between terminal voltage and field currents for each of the conditions of loading, using field current as abscissas.

No. 54. LIMITS OF REGULATION OF AN ALTERNATOR.

References. Thompson's "Dynamos," pp. 271 to 282; Karapetoff, pp. 523 to 525; Franklin and Esty, pp. 134 to 137; *Trans. Am. Inst. Elec. Eng.*, May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, D. B. Rushmore, The regulation of alternators; *Am. Elect'n*, August, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.*, October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.*, May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; April 22,

1904, H. M. Hobart and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.*, December 17, 1904, C. F. Guilbert, On the theory of the regulation of alternators; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rushmore, The regulation of alternators; *Elec. Wld. and Eng.*, September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.*, October 31 and November 28, 1903, B. A. Behrend, Testing of alternating current generators.

Object. To determine the limits between which the regulation of an alternator must come.

Theory and Method. The term regulation as applied to alternators means the percent., change in terminal voltage, as the load is decreased from normal value to no load, in terms of full load voltage. It is of course assumed, as in the case of separately excited direct current generators, that the field current and speed be maintained at the values required for full load normal voltage.

It is of value to the manufacturer to be able to determine the regulation of each machine before it is shipped. It is not possible, with the facilities available, to load large alternators to full non-inductive load before shipment. Hence some method of predetermination becomes valuable. Unfortunately, no simple and accurate method has been discovered. It is, however, easy to determine the limits between which the value of the regulation will fall. These have been called the *optimistic* and *pessimistic* limits. The terminal voltage differs from the generated voltage due to two causes, the armature impedance and the armature reaction. In the determination of the synchronous reactance it was found that these two effects could, in some cases, be considered as one. In Figure 54, curve OA is the open circuit saturation curve of an alternator.

The armature reaction and armature reactance may be combined and considered as a demagnetizing action on the field. Let $OR = m$ be the value of the field current for full load armature

current on short circuit. This would then be the value of the demagnetizing ampere turns of the armature for full load current and zero power factor. If the distance (m) be added to the abscissas of the open circuit saturation curve, and a curve drawn between these new points, the curve RC_1 will result. The length of the ordinate between these curves at normal voltage divided by normal voltage gives the optimistic value of the regulation.

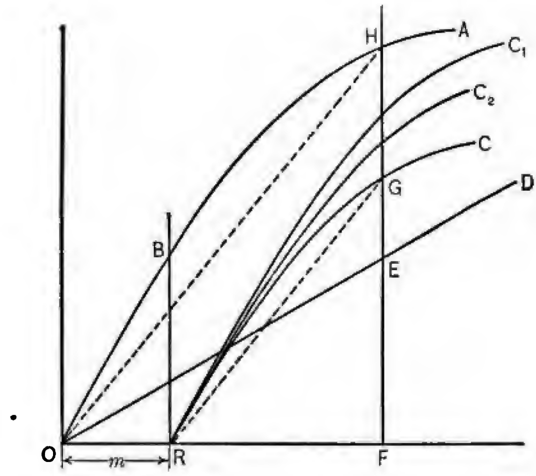

Fig. 54. Optimistic and pessimistic load saturation curves of an alternator.

The armature drop may also be considered as due to a reactance equal to the synchronous reactance. This reactance multiplied by the full load armature current would give the value of the reactance drop for this load and its corresponding field current. This will of course equal the value of the ordinate RB of the curve OA, which gives the point R common to both curves. Other points may be found in the following manner. Take some value of armature current, as EF, on a curve between short circuit current and field current, say three times full load current. The field current OF corresponds both to EF and to the voltage HF necessary to send this current through the armature. Lay off HG equal to $\frac{1}{3}$ HF on this ordinate, corresponding to full load current. G is then another point on the curve required.

Other points may be found in the same manner, giving the curve *RC*. An easier construction is the following. Draw the line *OH*. From *R* draw a line parallel to *OH* cutting *HF* at *G*. Other points on the curve may be found in the same manner. The value of the regulation derived from the curve *RC* is known as the pessimistic value.

The true value of the regulation at zero power factor may be derived from a curve lying somewhere between the curves *RC* and RC_1, as RC_2. Curves for other power factors may be constructed from the load saturation curve at zero power factor. Curves RC_1 and *RC* are known as the optimistic and pessimistic limits of regulation of the alternator.

Data. Take the data derived in Experiments 49 and 50. Determine the optimistic and pessimistic values of regulation for normal voltage and also for half normal voltage.

Curves. Derive curves similar to *RC* and RC_1 for normal load.

Suggestion. For the purpose of comparison, these curves should be made for the alternator used in the regulation experiments.

No. 55. CONSTRUCTION OF THE LOAD SATURATION CURVE OF AN ALTERNATOR AT ZERO POWER FACTOR.

References. Russell, Vol. 2, pp. 55 to 58; Thompson's "Dynamos," Vol. 2, pp. 275 to 276; Thomälen, p. 303; Karapetoff, pp. 518 and 519; Arnold, Vol. 4, pp. 347 and 636; *Am. Elect'n*, August, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.*, October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.*, May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; April 22, 1904, H. M. Hobart and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.*, December 17, 1904, C. F. Guilbert, On the theory of the

regulation of alternators; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rushmore, The regulation of alternators; *Elec. Wld. and Eng.*, September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.*, October 31 and November 28, 1903, B. A. Behrend, Testing of alternating current generators.

Object. To construct the load saturation curve of an alternator when operating at zero power factor. This curve is of value in determining the regulation at other power factors.

Theory and Method. The following method is an approximate one for constructing the load saturation curve of an alternator at zero power factor. It aims to separate the two components, armature reaction and inductive drop.

The armature ampere turns with lagging currents are subtracted directly from the field ampere turns. When the current lags 90 degrees behind the electromotive force, the reactance drop is in line with the induced electromotive force, and hence this drop is subtracted directly from the induced electromotive force to obtain the terminal electromotive force, as explained in the Introduction to Experiments on Synchronous Machines. Hence the reactance drop may be subtracted from the terminal electromotive force on open circuit.

In Figure 55*A*, *OA* is the open circuit saturation curve plotted with ampere turns as abscissas. The number of ampere turns in the armature corresponding to full load current are calculated from the design constants of the machine. Let these be equal to *KL*. The distance *OR* represents the field ampere turns necessary to send full load current through the armature on short circuit. *LR* is then the drop due to the reactance of the armature. The hypotenuse *KR* represents the total armature drop. If lines be drawn from *OA* parallel and equal to *KR*, as *K'R'*, a locus of their ends will give the curve RC_2, the load saturation curve of the alternator at zero power factor. From this curve a satura-

tion curve at other power factors may be constructed. This is usually known as Poitier's method.

When the value of the armature ampere turns is not known, and one point on the load saturation curve at zero power factor may be found by experiment, the following method may be used.

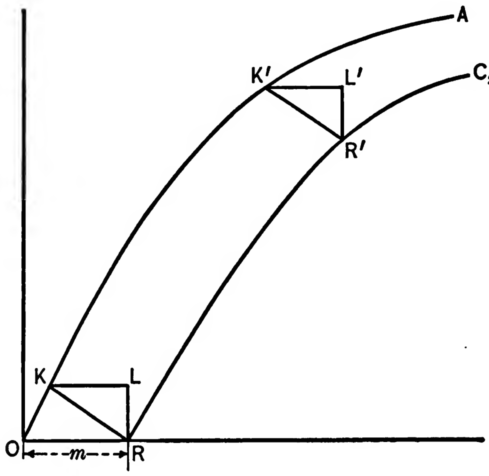

FIG. 55*A*. Construction of the load saturation curve of an alternator. Poitier's method.

Let R' be the point found by experiment. Find by trial the line $K'R'$ which may be taken equal and parallel to a line KR. Use this line to construct the curve RC_2 as before.

If both lines OA and RC_2 be found by test, the line $K'R'$ may be found by the following method. Trace the line RC_2. Move this traced line always parallel to itself, until the line RC_2 coincides as nearly as possible with OA. The shortest path travelled by any point, as R', gives the line $K'R'$.

From any of these constructions the values of the armature reaction and armature inductance may be determined. For $K'L'$ is the armature demagnetizing ampere turns, and $L'R'$ is the armature reactance drop from which the inductance may be calculated.

When a point such as R' is known on the load saturation curve

at zero power factor, the construction shown in Figure 55*B* may be used. From R' a line is drawn parallel and equal to RO, as $R'O'$. From O' a line is drawn parallel to the open circuit saturation curve at the point O, cutting the curve OA at K'. $K'R'$ then conforms with the same line as constructed by the other methods.

Curves. From data obtained in previous experiments plot the curve OA and locate the point R. By one of the methods ex-

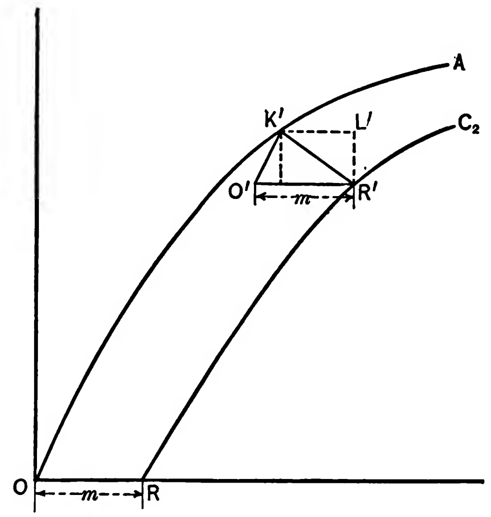

FIG. 55*B*. Construction of the load saturation curve of an alternator. Poitier's method.

plained above plot the curve RC_2, the load saturation curve at zero power factor. If data is obtainable for plotting RC_2 from experiment, check one method by another.

Compute. The values of the armature reaction and of the armature inductance by the method suggested above.

No. 56. THE TORDA-HEYMAN METHOD OF OBTAINING THE LOAD SATURATION CURVE OF AN ALTERNATOR AT ZERO POWER FACTOR.

References. Karapetoff, pp. 525 to 527; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, D. B. Rushmore, The regulation of alternators; *Elect'n Lond.*, April 22, 1904, Torda-Heymann, Theoretical basis for the experimental predetermination of the regulation of alternators; *Am. Elect'n,* August, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.*, October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.*, May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; April 22, 1904, H. M. Hobart and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.*, December 17, 1904, C. F. Guilbert, On the theory of the regulation of alternators; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rushmore, The regulation of alternators; *Elec. Wld. and Eng.*, September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.*, October 31 and November 28, 1903, D. A. Behrend, Testing of alternating current generators.

Object. To plot the load saturation curve of an alternator at zero power factor without loading the machine.

Theory and Method. This method makes use of a quantity called the apparent reluctance of the magnetic circuit. This is defined as the ratio of the magnetic flux on open circuit to the field ampere turns producing it, or the ratio of the open circuit terminal voltage to the field current as obtained from the open circuit saturation curve of the machine.

In Figure 56, let the curve *OA* be the open circuit saturation

curve of the alternator. Let OR be the field current necessary to send full load current through the armature on short circuit. If the reluctance of the magnetic circuit remains constant for all

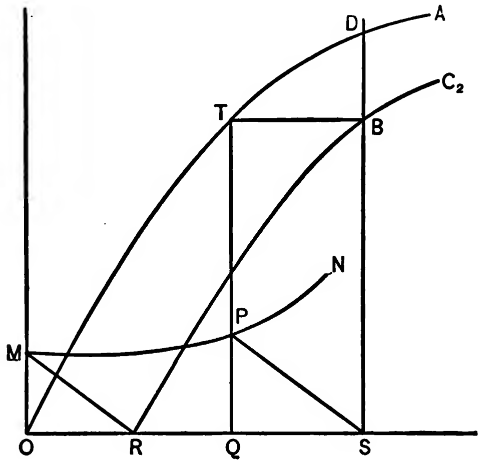

Fig. 56. Construction of the load saturation curve of an alternator. Torda-Heyman method.

values of field current, the distance OR could be added to each of the abscissas of the curve OA to produce the curve RC_2, as is done in the optimistic method explained in Experiment 54. For

$$OQ = OS - OR.$$

Instead of this simple relation, Torda-Heyman uses the following empirical formula:—

$$OQ = OS - OR\left(\frac{\mathcal{R}}{\mathcal{R}_S}\right)^2,$$

where $\mathcal{R}$ is the apparent reluctance at any value of field current, and $\mathcal{R}_s$ is the lowest value of reluctance obtained, when the fields are not saturated. From the ratio of the ordinates to the abscissas of the curve OA, the curve MN is drawn using these ratios squared as ordinates and field current as abscissas. Then

$$OM = \mathcal{R}_S^2.$$

Take any value of field current as OQ. Draw the ordinate

QT intersecting MN at P. Then

$$QP = \mathfrak{R}^2,$$

and

$$QS = OR\frac{QP}{OM}.$$

Draw the abscissa TB equal to QS. Then B is a point on the curve RC_2. From the above equation it may be seen that

$$\frac{QS}{QP} = \frac{OR}{OM}.$$

Hence, if MR be drawn, any line PS may be drawn parallel to it from the intersection of an ordinate, as QT, with MN at P. Having located S, the lines SD and TB may be drawn giving the points on RC_2. RC_2 is then the full load saturation curve at zero power factor. The saturation curve for other loads may be constructed in a similar manner. From these the saturation curves and external characteristics at other power factors may be constructed.

Curves. Derive the curve OA and the point R by some method as explained in Experiments 49 and 50. Calculate the values for the curve MN, and plot the curve. From these curves construct the curve RC_2.

No. 57. REGULATION OF AN ALTERNATOR FROM ITS OPEN CIRCUIT AND LOAD SATURATION CURVES.

References. Karapetoff, pp. 527 to 530; Esty, pp. 164 to 169; Standard Handbook, Sec. 7, Art. 280 to 289; Arnold, Vol. 4, pp. 347 to 350; *Am. Elect'n,* August, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.,* October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.,* May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; April 22, 1904, H. M. Hobart

and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.*, December 17, 1904, C. F. Guilbert, On the theory of the regulation of alternators; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rushmore, The regulation of alternators; *Elec. Wld. and Eng.*, September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.*, October 31 and November 28, 1903, B. A. Behrend, Testing of alternating current generators.

Object. To predetermine the regulation of an alternator from its open and short circuit tests.

Theory and Method. In this method the necessary tests are those performed in Experiments 49 and 50. The method holds very well for alternators that are worked on the straight portion of the saturation curve, but it is inaccurate for most commercial machines.

In Figure 57*A*, let *OA* represent the open circuit saturation

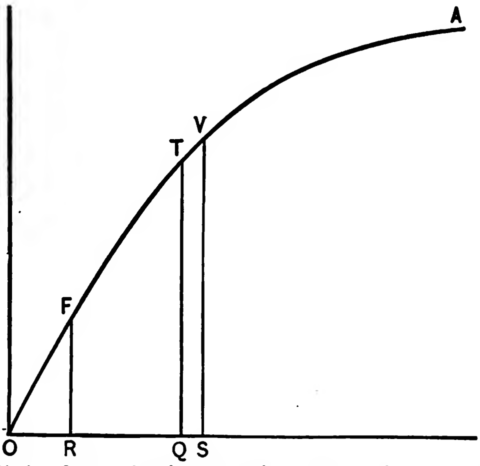

FIG. 57*A*. Open circuit saturation curve of an alternator.

curve of an alternator. Let *OR* be the field current necessary to produce full load current on short circuit and *RF* the open circuit voltage corresponding to it. Let *QT* be the normal

voltage on open circuit, and OQ the field current necessary to produce it. Then, if OT is a straight line,

$$\frac{OR}{OQ} = \frac{RF}{QT},$$

or, the field currents are in direct ratio to the voltages. Construct the voltage diagram for non-inductive load, Figure 57*B*,

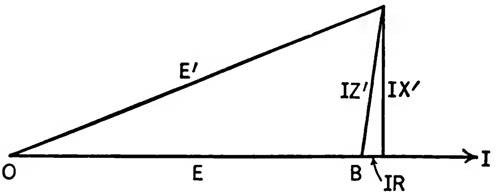

FIG. 57*B*. Voltage diagram of an alternator. Non-inductive load.

using the values of armature resistance and synchronous reactance as previously found, and E, the value of the normal voltage. The generated voltage E' corresponds to the open circuit voltage with a field current necessary to produce the terminal voltage E under full non-inductive load. Or, by geometry,

$$E' = \sqrt{(E + IR)^2 + \overline{IX'}^2}. \qquad (57a)$$

As IR is small compared with IX' this may be written

$$E' = \sqrt{E^2 + \overline{IX'}^2}, \qquad (57b)$$

or, substituting the values of field current for voltage,

$$OS = \sqrt{\overline{OQ}^2 + \overline{OR}^2}, \qquad (57c)$$

where

I_{f_0} OQ = field current to produce normal voltage on open circuit,
I_f OR = field current to produce normal current on short circuit; and
I_f'' OS = field current to produce normal voltage on full non-inductive load.

Equation 57*c* is a familiar expression with designing engineers.

It is shown graphically in Figure 57C and is a rough and easy way to check more exact methods. The regulation of the machine is then

$$\frac{E' - E}{E}.$$

The ordinate SV in Figure 57A should equal E'. Equation (57c) may be stated as follows. *The field current necessary to produce normal voltage on full non-inductive load is the square root of the sum of the squares of the field current to produce normal voltage on open circuit and the field current to produce full load current on short circuit.* It is customary to calculate the field current OS and to find the voltage corresponding to this from the open circuit saturation curve.

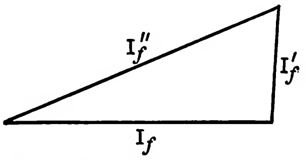

Fig. 57C. Field current diagram of an alternator.

The regulation at other power factors may be found in a similar manner. A convenient method of construction is to draw a semi-circle with radius equal to the normal voltage to some convenient scale as shown in Figure 57D. Divide the radius OF into 100 equal parts to represent the power factor of the load. Take any position of the radius as OB, where OA is the power factor. Draw the IR and IX drop lines in phase with and at right angles to OF, the current axis. Complete the triangle OBH. Then OH is the generated electromotive force E'', and the regulation is

$$\frac{E'' - E}{E}.$$

A similar construction may be made in the quadrant OFC for capacity loads of any power factor. Such a construction is shown in $OB'H'$, where OH' is equal to E''' and the regulation is

$$\frac{E''' - E}{E}.$$

The *external characteristic* of a generator is the curve between the terminal electromotive force and the load current. In taking the external characteristic of a separately excited machine, the

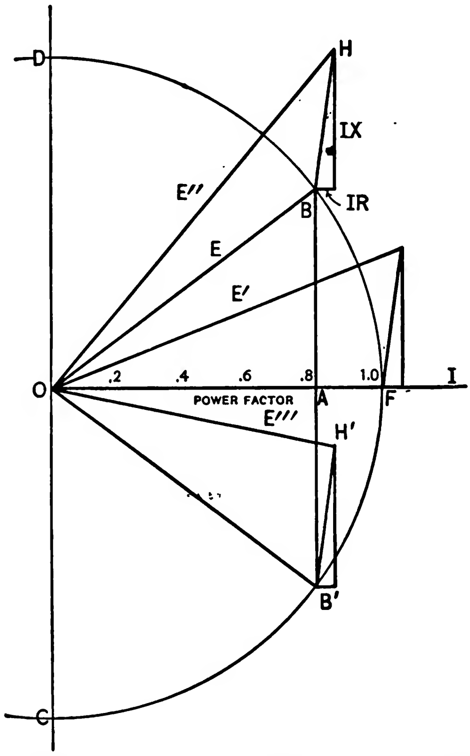

FIG. 57*D*. Voltage diagram of an alternator for any power factor.

field current is adjusted to give normal voltage at full load and normal frequency. The load is then reduced to zero, keeping the field current and the frequency constant. Readings of load current and terminal voltage are taken at frequent intervals. The external characteristic is then plotted from these readings. If inductive or capacity load is used, the power factor should be kept constant.

These curves may be derived from the construction in the following manner. In Figure 57*E*, similar to Figure 57*B*,

divide the IZ drop BH into four equal parts to represent the

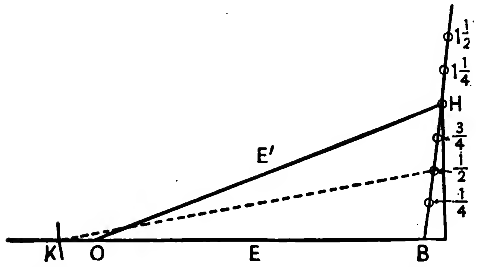

FIG. 57*E*. Determination of the external characteristic on non-inductive load.

impedance drop on one fourth, one half, three fourths and full load. Extend the same line and lay off equal parts to represent the impedance drop on one and one fourth and one and one half load. With each of these points of division as a center and with a radius equal to E', cut the line BO produced, as at K, for one half load. The distance BK is the terminal voltage for that load. Other values may be found in a similar manner, and the external characteristic APD, Figure 57F, may be constructed.

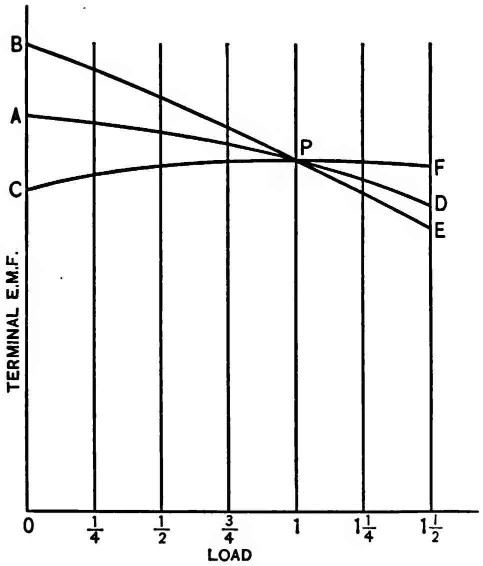

FIG. 57*F*. External characteristics of an alternator. Non-inductive, inductive and capacity loads.

A similar construction may be made for inductive and capacity loads of a given constant power factor. In this case the lines BO and $B'O$, Figure 57D, should be produced and E'' and E''' used as radii. The curves should be plotted as BPE and CPF in Figure 57F. These lines will pass through the common point P at normal voltage and full load.

Data. The open circuit saturation curve and the synchronous impedance should be derived as in Experiments 49 and 50.

Calculate. The regulation curves for non-inductive load and for inductive and capacity loads of 80 percent. power factor.

Curves. Plot the external characteristics for non-inductive load and for inductive and capacity loads of 80 percent. power factor.

Suggestion. For the purpose of comparison, these curves should be made for the machine used in other experiments on regulation.

Questions. In the construction used in this experiment it is assumed that the full load terminal pressure is the same at all power factors. Will the field heating be the same? The armature heating? The size of the exciter required? Assuming the same exciter voltage, would you design an alternator for operation on inductive load of greater or less field resistance than on capacity load?

Is a variable reactive load of any kind suitable in general on a lighting circuit? Assume that the alternator field is adjusted to give normal voltage at full load unity power factor. The armature current is held constant and the power factor is changed to 80 percent. lead and then to 80 percent. lag. What is the terminal voltage variation? What is the maximum current output of the machine for each power factor considered?

No. 58. DETERMINATION OF THE REGULATION OF AN ALTERNATOR.

References. *Elec. Wld. and Eng.*, September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Am. Elect'n*, Au-

gust, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.*, October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.*, May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; April 22, 1904, H. M. Hobart and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.*, December 17, 1904, C. F. Guilbert, On the theory of the regulation of alternators; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rushmore, The regulation of alternators; *Elec. Wld. and Eng.*, September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.*, St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.*, October 31 and November 28, 1903, B. A. Behrend, Testing of alternating current generators.

Object. To determine the regulation of an alternator without loading the machine.

Theory and Method. A fairly accurate but somewhat complicated method of determining the regulation of an alternator without loading the machine has been described by C. E. Canfield (see references). It involves measuring the inductance of the armature with the fields removed. The value of inductance measured in this way is correct for heavily inductive loads, but it is usually too small for non-inductive loads. The value of regulation as found by this method agrees very closely with that obtained by actually loading the machine.

The open circuit saturation curve and the short circuit characteristic are obtained as in Experiments 49 and 50. The field is then removed from the armature and the inductance of the armature coils measured with alternating current of rated frequency, as in Experiment 1.

The effect of armature reaction with the armature on short circuit may be determined in the following manner. From the curve between armature current and field current on short circuit

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the value of field current corresponding to full load armature current is determined. The value of electromotive force corresponding to this field current is found from the open circuit

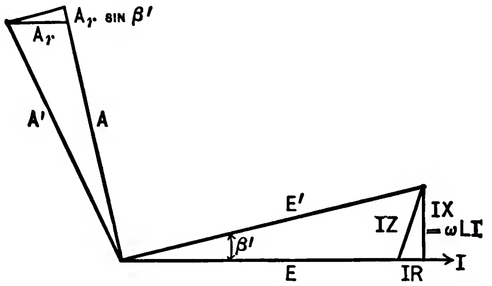

FIG. 58*A*. Emf. and mmf. diagram of an alternator. Non-inductive load.

saturation curve. From this value the reactance drop ωLI is subtracted and the field current A_r corresponding to this electromotive force is determined. Then A_r corresponds to the armature reaction on short circuit. If the curve is plotted with ampere turns instead of field current, the result will be in ampere turns.

The armature reaction at any other power factor may be determined in the following manner. Referring to Figure 58*A*, which is for the case of non-inductive load on the machine, the

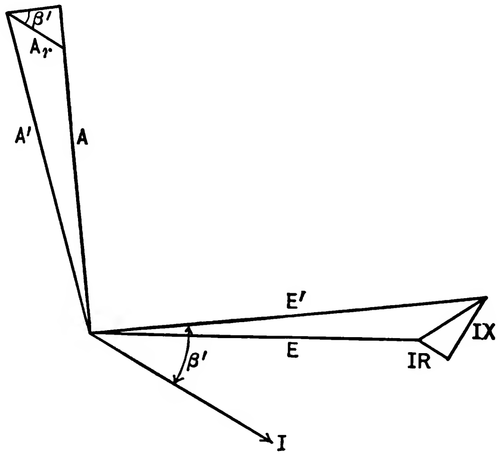

FIG. 58*B*. Emf. and mmf. diagram of an alternator. Inductive load.

resistance drop IR and the reactance drop ωLI are added to the terminal electromotive force E to determine the generated electromotive force E'. To produce this electromotive force E' on open circuit the field current A would be required. This is laid off at right angles to E' in the direction of the flux. To this must be added the armature reaction A_r in its proper phase relation. A_r is laid off in phase with the armature current from the end of A. The resultant of A and A_r is the field current A' necessary to produce the electromotive force E at the terminals of the machine under full non-inductive load.

From the open circuit saturation curve, the voltage corresponding to A' is E'' (not shown in the diagram). Then

$$\frac{E'' - E}{E}$$

is the regulation of the machine.

The construction for inductive loads is shown in Figure 58*B* and for capacity load in Figure 58*C*. Here the current is laid

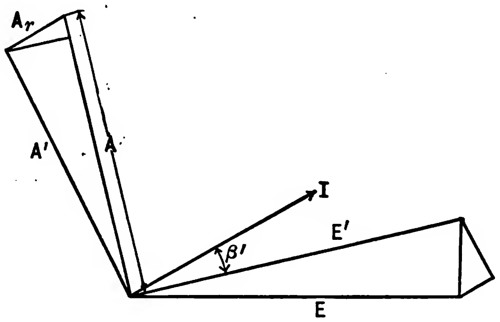

FIG. 58*C*. Emf. and mmf. diagram of an alternator. Capacity load.

off at the angle β with E, determined by the power factor of the load. The IR and IX drops are added in their proper phase relations to the current. From this construction, E' and the angle β' are determined. The rest of the construction is similar to that for non-inductive load.

The regulation may also be found without the graphical con-

struction in the following manner. For non-inductive load

$$E' = \sqrt{(E + IR)^2 + (\omega LI)^2}. \qquad (58a$$

From E' the value of A is determined. A_r must also be determined as before. Then

$$\sin \beta' = \frac{\omega LI}{E'},$$

$$\cos \beta' = \frac{E + IR}{E'},$$

and

$$A' = \sqrt{(A + A_r \sin \beta')^2 + (A_r \cos \beta')^2}. \qquad (58b)$$

From A' the value of E'' and the regulation of the machine may be determined.

For inductive load

$$E' = \sqrt{(E \cos \beta + IR)^2 + (E \sin \beta + \omega LI)^2},$$

$$\sin \beta' = \frac{E \sin \beta + \omega LI}{E'},$$

$$\cos \beta' = \frac{E \cos \beta + IR}{E'},$$

and equation (58b) applies for the value of A'.

Data. Take data the same as for Experiments 49 and 50. Remove the field structure and measure the armature inductance as in Experiment 1.

Compute. The regulation of the alternator for various loads up to 50 percent. overload, with non-inductive load and with inductive and capacity loads of 80 percent. power factor.

Curves. Draw regulation curves for the three kinds of load.

Suggestion. For the purpose of comparison, these curves should be made for the alternator used in other regulation experiments.

No. 59. REGULATION OF AN ALTERNATOR AT ANY POWER FACTOR FROM THE CURVE AT ZERO POWER FACTOR.

References. Karapetoff, pp. 528 and 529; *Am. Elect'n,* August, 1901, A. S. McAllister, The regulation of alternating current generators; *Elec. Wld. and Eng.,* October 18, 1902, V. D. Moody, Computation of regulation of alternating current generators; *Trans. Am. Inst. Elec. Eng.,* May 19, 1903, B. A. Behrend, The experimental basis for the theory of the regulation of alternators; April 22, 1904, H. M. Hobart and F. Punga, A contribution to the theory of the regulation of alternators; *Elec. Wld. and Eng.,* December 17, 1904, C. F. Guilbert, On the theory of the regulation of alternators; *Trans. Int. Elec. Cong.,* St. Louis, September, 1904, Vol. 1, pp. 729 to 761; D. B. Rushmore, The regulation of alternators; *Elec. Wld. and Eng.,* September 3, 1904, C. E. Canfield, Calculation of alternator regulation; *Trans. Int. Elec. Cong.,* St. Louis, September, 1904, B. A. Behrend, The testing of alternating current generators; *Elec. Wld. and Eng.,* October 31 and November 28, 1903, B. A. Behrend, Testing of alternating current generators.

Object. To be able to apply the load saturation curves at zero power factor to the determination of the regulation of alternators at other power factors.

Theory and Method. In this experiment the open circuit saturation curve is plotted from data and a load saturation curve constructed by any one of the methods described in Experiments 53, 55 and 56. To find the regulation at other power factors the following construction due to Kapp is used. In the Introduction to Experiments on Synchronous Machines, it was pointed out that, with the machine operating at zero power factor, the armature resistance drop can be omitted and the armature reactance drop taken in phase with the terminal electromotive force. In Figure 59*A* are shown the open circuit saturation curve and the load saturation curve of an alternator at zero power factor. In

Figure 59*B* the phase relations are shown graphically. HC is taken equal to FA, and HD is taken equal to FC_2. Then DC, the reactance drop, will be equal to AC_2. For any other power

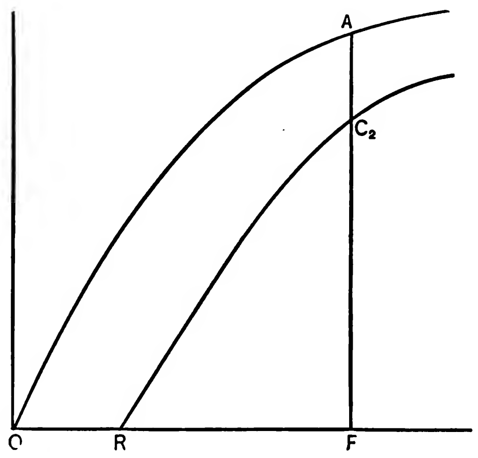

Fig. 59*A*. Open circuit and load saturation curves of an alternator.

factor, DC' is laid off at an angle equal to $(90° - \beta)$ with the line HD, $\cos \beta$ being the power factor of the load. With C' as a

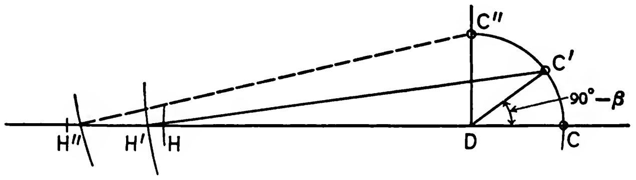

Fig. 59*B*. Kapp diagram for determination of the regulation of an alternator at any power factor from the value at zero power factor.

center and AF (C'H') as radius, the line DH is cut at H' giving DH', the terminal electromotive force. Regulation is then

$$\frac{\cancel{CH} \; (C'H') - DH'}{DH'}.$$

C'H' = E'
DH' = E
DC' = IZ'

The triangle $DC''H''$ shows the construction for non-inductive load.

Data. Obtain data for the construction of curves as explained in Experiment 53, 55 or 56.

Compute. The regulation curve of the alternator for non-inductive load and also for loads of other power factors.

Curves. Plot regulation curves for non-inductive load and for inductive and capacity loads of 80 percent. power factor. Plot a curve showing the regulation of the alternator for various power factors, using power factor as abscissas. Plot a curve between terminal voltage and power factor for the machine operating at full load current.

Suggestion. For the purpose of comparison, these curves should be made for the alternator used in other experiments on regulation.

No. 60. EFFICIENCY OF AN ALTERNATOR BY THE RATED MOTOR METHOD.

References. Swenson and Frankenfield, Vol. 1, Experiment 72; Fleming, Vol. 2, p. 535.

Object. The object of this experiment is to determine the efficiency of an alternator by direct measurement of the intake and output.

Theory and Method. In this method the output is measured electrically, and the intake is measured by means of a rated motor. This motor should preferably be a shunt machine, rated as in Direct Current Experiment 72. It may, however, be an alternating current motor, but here the difficulty lies in regulating the speed, unless one can control the speed of the prime mover from which the motor gets its power.

The alternator should be driven at the speed for each load which would be given by its prime mover when permanently installed. This is not usually feasible, and a constant speed may be held throughout the test.

If the current be leading or lagging, the output should be measured by means of wattmeters. If the rated motor is an alternating current machine, its intake should also be measured

by means of wattmeters. It is preferable to direct connect the motor and generator by means of a flexible coupling, otherwise allowance must be made for belt loss which is difficult to measure.

Data. Drive the generator by means of a rated motor of sufficient capacity. Apply a non-inductive load to the alternator, and take readings of current, pressure and speed of both motor and generator, from no load to an overload, being sure to adjust the speed and pressure according to the normal running conditions. Measure the power absorbed in the field of the alternator.

Calculate. The efficiency of the alternator for the various loads applied.

Curves. Plot an efficiency curve for the alternator, using output as abscissas. Plot all the curves used in determining the output of the rated motor.

Questions. Do you consider this an accurate method? Give reasons. What do the Standardization Rules of the American Institute of Electrical Engineers state concerning exciter losses?

No. 61. EFFICIENCY OF AN ALTERNATOR BY THE STRAY POWER METHOD.

References. Swenson and Frankenfield, Vol. 1, Experiment 68; Esty, pp. 174 to 180; Karapetoff, p. 509; Standard Handbook, Sec. 7, Art. 252 to 279; Lamb, pp. 136 to 139; Arnold, Vol. 4, pp. 638 and 639.

Object. To determine the losses of an alternator under all conditions of loading, and to use these losses in calculating the efficiency of the machine. This method economizes power.

Theory and Method. The stray power method of testing alternators is, in general, similar to the direct current method explained in Direct Current Experiment 68. The losses in the machines are: (*a*) Friction and windage, (*b*) I^2R losses in the armature conductors and in the field winding, (*c*) core losses and (*d*) load losses. Loss (*d*) will be described later.

Knowing the synchronous impedance and the armature re-

sistance, together with the open circuit saturation curve as obtained in Experiments 49 and 50, the field necessary to produce normal pressure at any given load may be calculated. From this data the field loss may be calculated for the same load.

To measure the open circuit core loss and the friction and windage, the machine is driven by means of a rated motor. The power required to drive the alternator at normal speed and with no field excitation is first determined. If the machine is in its own bearings, this loss is to be charged against the machine as friction and windage, and is assumed constant for all loads. Next is determined the additional power required to drive the alternator on open circuit with various field excitations from zero to about 25 percent. above normal. At frequent intervals during this test friction and windage readings should be taken for the purpose of checking the first determination. This is to see that the friction and windage has not changed enough to affect the accuracy of the other readings.

The armature is now short circuited (the fields being unexcited) and the power required to drive the alternator with various armature currents from zero to about 50 percent. over load current, is determined; the field current necessary being adjusted to give the proper values of armature current. This loss includes friction and windage, I^2R loss in the armature conductors, eddy current loss in the armature conductors, iron losses in the armature core, eddy current loss due to excessive leakage flux in the yoke and end leads, etc. This latter loss is much greater on short circuit, which is equivalent to about zero power factor, than it would be on non-inductive load, because the armature ampere turns directly oppose the field ampere turns. These losses, with the exception of the friction and windage, and the I^2R loss, are known as the load losses. The Standardization Rules of the American Institute of Electrical Engineers recommend that, in calculating the efficiency on non-inductive load, one third of these load losses be used.

The losses in the machine for any load are then the following:

1. Friction and windage,
2. I^2R loss in the armature,
3. I^2R loss in the field,
4. Open circuit iron loss, and
5. One third of the load losses.

The friction and windage losses are neglected if the machine is to be direct connected to a prime mover.

Data. Run the machine for some time, to get the oil evenly distributed in the bearings and the temperature constant. Measure the friction and windage loss at normal speed. Measure the open circuit iron loss at various voltages from zero to about 25 percent. above normal. Measure the short circuit losses with various armature currents from zero to about 50 percent. above normal. Measure the resistance of the armature and field at normal temperature. Carefully check the friction and windage loss during the test to see if it has changed appreciably.

Calculate. The efficiency of the alternator for normal voltage at one fourth, one half, three fourths, full load and 25 percent. overload.

Curves. Plot a curve between open circuit core loss and

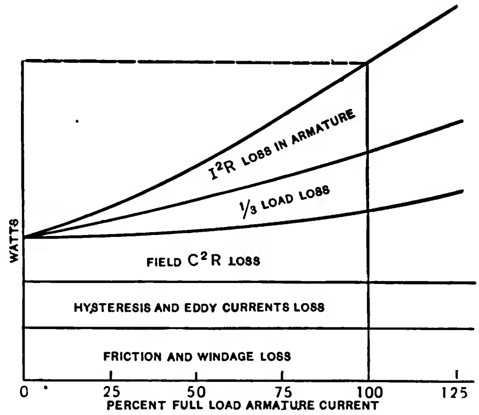

FIG. 61. Curves between losses and output, for an alternator.

terminal voltage, using voltage as abscissas. Plot a curve between short circuit ~~core~~ loss and armature current, using arma-

ture current as abscissas. Plot a set of curves as in Figure 61, using values of armature current as abscissas, the last one giving the total losses as ordinates. Plot a curve between efficiency and percent. load, using percent. load as abscissas.

Questions. Would you expect a higher or a lower efficiency for a given armature current on an inductive or a capacity load than that obtained on a non-inductive load? Why? Would the efficiency for a given power factor be higher or lower for an inductive load than for a capacity load of the same amount? Why?

No. 62. TEMPERATURE AND EFFICIENCY TESTS OF AN ALTERNATOR BY THE DIFFERENTIAL EXCITATION METHOD.

References. Karapetoff, pp. 512 to 515; Fleming, Vol. 2, pp. 581 to 585; *Proc. Inst. Elec. Eng.*, Vol. 22, 1893, pp. 116 to 136, W. M. Mordey, On testing and working arternators.

Object. To operate an alternator under normal conditions as to heating in the field and armature, with the consumption of the minimum amount of power in the driving motor, and to measure its efficiency under these conditions.

Theory and Method. If the field poles of an alternator having a single path armature are divided into two equal groups and these so magnetized as to oppose each other, Figure 62*A*, there

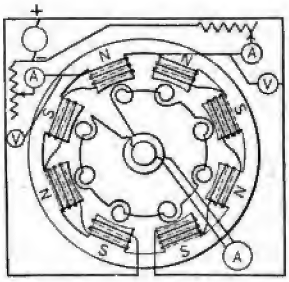

Fig. 62*A*. Connections for differential excitation of an alternator. Field divided into two paths.

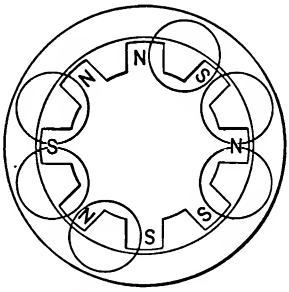

Fig. 62*B*. Paths of flux for an eight pole alternator with differential excitation.

will be two equal and opposite electromotive forces generated in the armature and, consequently, no terminal pressure if the flux is the same in the two groups. It will be assumed, in the preliminary discussion, that there has been no change in the distribution of the flux caused by the new method of arrangement of the field circuit. Later it will be shown how the flux is altered and what the effect is upon the value of the test.

Suppose the armature is short circuited through an ammeter and that one half of the field receives a slightly greater excitation than the other. If the alternator be run at normal speed a current will flow in the armature equal to

$$I = \frac{E_1 - E_2}{\sqrt{R^2 + X^2}},$$

where E_1 and E_2 represent the armature electromotive forces and R and X represent the armature resistance and reactance respectively. Very little power will have to be supplied to the machine, even with full load armature current, since the weaker half of the field causes a motor action and the stronger half a generator action. The difference in power represents the losses in the machine for that particular value of armature current. This power is best supplied by means of a rated motor. By adjusting the relative excitation of the halves of the field (the mean excitation remaining normal) a current of any desired value may be caused to circulate in the armature. Let

$I =$ the armature current,
$E =$ the normal terminal pressure of the armature,
$W_f =$ the watts supplied the field under normal operating conditions, and
$W_e =$ watts supplied the alternator by the rated motor.

Then the efficiency is

$$\eta = \frac{IE}{IE + W_f + W_e},$$

for the condition of non-inductive load on the alternator. In the

case of an inductive load of power factor (p), the efficiency is

$$\eta = \frac{pIE}{pIE + W_f + W_e},$$

but the value of W_f is the field loss for normal operation at the required power factor.

The effect of the changed condition of the field may now be considered. Figure 62*B* shows the new condition of the field for an 8-pole machine, while Figure 62*C* represents the condition for one with 10 poles. In either case the distribution of flux is radically different from normal conditions. At normal excitation the flux is considerably reduced under adjacent like poles and increased under the poles adjoining these. The effect is to increase the hysteresis and eddy current losses, and also to alter to some extent the form of the pressure wave of the alternator.

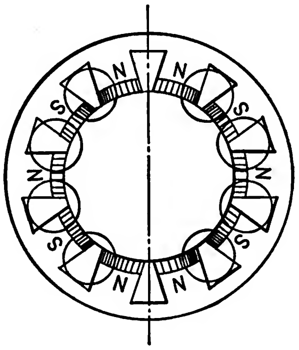

Fig. 62*C*. Paths of flux for a ten pole alternator with differential excitation.

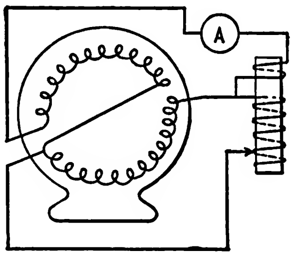

Fig. 62*D*. Ayrton's modification of Mordey's test.

The greater the number of poles the less this rearrangement will affect the total result. Thus the effect on a machine having 24 or more poles would probably be within the limit of error in measurement, while with an 8-pole machine this effect would be enough to make the results entirely unreliable in most cases. A further objection to this method is that an unbalanced magnetic pull is exerted upon the armature, because one group of poles is

more strongly magnetized than the other. This is liable to cause serious vibration in the whole structure. This effect may be overcome by dividing the fields into several groups spaced around the field structure. Another difficulty encountered in using this method, which is equally serious no matter what the number of the poles, is due to the fact that the current does not have the same phase angle with respect to the generated pressure which it has when the machine is running under normal conditions. In all cases of normal operation the current must be

$$I = \frac{E'}{\sqrt{(R+r)^2 + (X+x)^2}},$$

where

E' = the total generated pressure,
R = the external resistance,
r = the armature resistance,
X = the external reactance, and
x = the armature reactance.

The phase angle of the current with respect to the generated pressure is

$$\beta = \tan^{-1}\frac{X+x}{R+r}.$$

Therefore, the current has the same phase relation to the generated pressure in the test as it has under normal conditions only when

$$\frac{X}{R} = \frac{x}{r},$$

a condition which would occur but infrequently. Since x/r is generally large, the armature current would lag behind the generated pressure considerably more in the test than it would under normal conditions. The effect would be, as far as the generator part of the armature is concerned, to increase the angle of current lead. The effect in both cases is to increase the armature reaction above normal for each load considered, affecting the wave form of the generated pressure and the losses due to

hysteresis and eddy currents. This method of testing is far from being an ideal one as regards accuracy, although it possesses the advantage of economy of power. The general effect of the changed conditions would be to give values of efficiency which are too low, since the friction loss remains practically normal and the hysteresis and eddy current losses are increased.

This method is Ayrton's modification of a method due to Mordey which was first used on the Mordey alternator. The armature being stationary, the coils were divided into circuits which were arranged to oppose each other, being also short circuited through an ammeter. Since there is but one field coil in this type of machine, the necessary difference in pressure in the two parts of the armature is obtained either by dividing the armature coils unevenly or by increasing the pressure in one part of the armature by means of a transformer arranged as in Figure 62*D*. In either case, if the field excitation is maintained normal, it is necessary to arrange to vary the armature current either by the introduction of resistance or inductance or by the use of a variable ratio transformer as in Figure 62*D*. Although especially adapted to the Mordey alternator, on account of its low armature reaction, it is open to the objections given above for Ayrton's method and has several additional disadvantages. Many alternators have armatures so wound that it would be impracticable to make connections as desired in the test. In armatures designed for high voltages it is undesirable to disturb the insulation of the winding to make these connections.

A method used in large machines with revolving field and a sufficient number of poles, is the following. With the large number of poles it becomes necessary to use several pairs of active poles. The number of opposed (or "bucked") poles is calculated so that the active poles will furnish just enough pressure, when normally excited, to give full load armature current on short circuit. The synchronous impedance at full load, Experiment 50, is obtained experimentally, and the field current for full load armature current is represented as in Figure 62*E*. The

open circuit saturation curve is shown in Figure 62*F*, where *P* represents the point of normal pressure equal to *OE* volts. The field current *OC'*, Figure 62*F*, for full load armature current, is

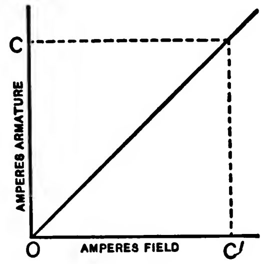

FIG. 62*E*. Curve between armature current and field current for an alternator, armature on short circuit.

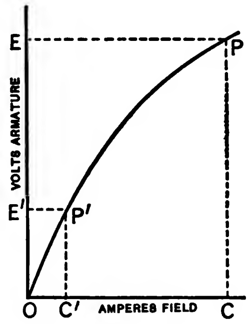

FIG. 62*F*. Open circuit saturation curve of an alternator.

laid off equal to *OC'* in Figure 62*E* and the point *P'* is obtained by drawing the ordinate *C'P'* to the saturation curve. This ordinate, which is *OE'*, is the pressure which would be generated by the field current *OC'* under no load conditions. The number of active poles is then obtained from the relation

$$\frac{\text{active poles}}{\text{total poles}} = \frac{OE'}{OE}.$$

If the machine has, say, 32 poles and the ratio of normal volts to volts in the armature on short circuit is 4 to 1, there would be 8 active poles; that is, 20 poles should be opposed to the remaining 12 poles. In order to equalize the magnetic pull this is usually done by "bucking" in two groups. This precaution is very necessary in testing large alternators of the revolving field type. This method is used by manufacturers principally for heat runs on large revolving field alternators.

The method of splitting the field as first described in this experiment is also largely used in heat runs of large revolving field alternators and has been developed to a point of refinement

by Behrend. To do this the middle point of the field circuit is grounded to the spider and a brush is mounted to bear on the shaft. This forms the common line to the two field circuits.

Data. Obtain data as in Experiments 49 and 50. Split the field as first described, or, if the other method is used, compute the number of active poles necessary to produce full load armature current on short circuit with normal field current. Drive the alternator by means of a rated motor. Connect the alternator field coils as computed for the desired result. Vary the field current of the machine and, if possible, put full load current on the machine until it attains a constant temperature. Shut down the machine and take the temperatures of the various parts and the resistances of the armature and field coils when hot. These resistances should also be taken when the machine is at room temperature.

Calculate. The efficiency of the machine from the data obtained. The rise in temperature from the resistance readings.

No. 63. DETERMINATION OF THE MOMENT OF INERTIA OF THE ROTATING PARTS OF A MACHINE.

References. Russell, Vol. 2, p. 169; Karapetoff, pp. 398 to 403; Lamb, pp. 139 to 143; Standard Handbook, Sec. 7, Art. 258 to 261; Arnold, Vol. 4, pp. 639 to 645; *School of Mines Qr.*, January, 1905, F. W. Seringhouse, The retardation method of measuring losses in motors and generators.

Object. In the design of synchronous machines, either for parallel operation as generators or as motors, the value of the moment of inertia is useful. It is also necessary to know the moment of inertia in order to determine the size of starting apparatus to use with motor-generator sets and similar machines which do not start under load. Here the entire load on the starters and starting motors is accelerating load, which is determined by the moment of inertia and the time of acceleration.

Theory and Method. In calculations concerning electrical machines, it often becomes highly important to know the moment of inertia of the rotating parts. It is difficult to calculate this accurately from the dimensions of the machine. When the machine is assembled it is difficult to obtain proper dimensions for computing this quantity. The following is an experimental method for its determination.

Let the motor slow down from an angular velocity ω (radians per second) to an angular velocity ω_1, in (t) seconds. If $d\omega/dt$ be the rate of change of angular velocity during this period and K be the moment of inertia of the rotating part, then

$$K\frac{d\omega}{dt} = \text{the retarding couple.}$$

In order to determine K, a known retarding force is applied by means of a friction brake. Let this force be F in c.g.s. units and let $(d\omega/dt)_1$ be the new rate of change at the same speed. Then

$$F = K\left[\left(\frac{d\omega}{dt}\right)_1 - \frac{d\omega}{dt}\right],$$

or

$$K = \frac{F}{\left(\frac{d\omega}{dt}\right)_1 - \frac{d\omega}{dt}},$$

where K is the moment of inertia in the same units in which F was measured.

The experiment is performed in the following manner. Drive the machine at a speed somewhat above normal by means of some outside source. Disconnect the driving source and allow the machine to slow down. Readings of speed are taken at short intervals (5 seconds) as the machine slows down. A brake is now applied to a pulley on the shaft of the machine and a small retarding torque applied and measured. The machine is again allowed to slow down under this new condition, readings being taken as before. Curves are plotted, covering both sets of read-

ings, between angular velocity and speed. From these curves the values of $d\omega/dt$ and $(d\omega/dt)_1$ are obtained at the desired speed, and K computed. It is well to repeat with two or more different loads, as a check upon this result.

This method is limited to machines having a large moment of inertia, because of the difficulty in obtaining enough readings for accurate work where the time of slowing down is short. Readings should not be taken at a very low speed as the friction coefficient increases rapidly just before the rotor comes to rest. If the windage is appreciable, the interval of time between $(d\omega/dt)_1$ and $d\omega/dt$ should be made short because a constant decrement of speed is assumed.

This difficulty may be overcome by mounting a flywheel, whose moment of inertia is known, upon the shaft of the machine. The moment of inertia of the combined system may then be determined and from the data at hand the moment of inertia of the machine alone may be computed.

Data. The rotating part may be driven from some external source, at a speed somewhat above normal. The driving source should then be disconnected and readings of speed taken by a tachometer at carefully timed intervals, as the machine slows down. Repeat, applying an additional retarding torque of measured value, by means of a brake.

Calculate. The value of ω for each speed and determine $d\omega/dt$ at normal speed from curves between ω and t. Compute the value of K.

Curves. Plot curves between ω and t for each condition.

Question. A motor-generator set operated from its direct current end requires an intake of 160 horse-power at full load and an average of 50 horse-power is required to accelerate it from rest up to full speed in 20 seconds. The field current is 7 amperes at 240 volts. Using a standard 50 horse-power starting box, what modification would you require to be made in the current capacities of the last contact and the release magnet before installation?

No. 64. DETERMINATION OF THE STRAY POWER LOSSES IN AN ALTERNATOR BY THE RETARDATION METHOD.

References. Bedell, Vol. 2, p. 169; Karapetoff, pp. 398 to 403; Lamb, pp. 139 to 143; Standard Handbook, Sec. 7, Art. 258 to 261; Arnold, Vol. 4, pp. 639 to 645; *School of Mines Qr.*, January, 1905, F. W. Seringhouse, The retardation method of measuring the losses in motors and generators.

Theory. In Experiment 63 a method was employed for finding the moment of inertia of the moving parts of a machine. This method may be extended into one for finding losses in the following manner.

If $K\, d\omega/dt =$ the retarding couple due to friction, K being measured in c.g.s. units. Then

$$K\omega \frac{d\omega}{dt} \times 10^{-7} = \text{the watts lost in friction.}$$

The value of K may be determined as in Experiment 63. If, now, the field of the machine is normally excited, an additional retardation, corresponding to the loss due to hysteresis and eddy currents in the iron, will occur. Let $(d\omega/dt)_2$ be the new value of rate of change of angular velocity at normal speed. Then

$$K\omega \left[\left(\frac{d\omega}{dt} \right)_2 - \left(\frac{d\omega}{dt} \right) \right]$$

will be the watts lost in the iron of the armature at normal speed and normal field. This method is of little value except on machines of fairly large moment of inertia.

Data. Find the value of K as outlined in Experiment 63. Apply normal field to the machine and, starting with a speed above normal, read the speed at definite intervals as the machine slows down. It is desirable to repeat for values of excitation somewhat above and somewhat below normal, as a check upon this result.

Curves. Plot curves similar to those in Experiment 63, for the determination of $d\omega/dt$.

Compute. The friction loss, the core loss and the efficiency of the machine and draw the efficiency curve.

Questions. What are the laws of variation of hysteresis torque, eddy current torque and windage torque, with speed, assuming constant magnetization? Why should the change in $d\omega/dt$, as applied in the equation, be taken over a short interval of time?

No. 65. THE PARALLEL OPERATION OF ALTERNATORS. (SYNCHRONIZING.)

References. Russell, Vol. 2, Chap. 7; Arnold, Vol. 4, Chap. 18 and 19; Standard Handbook, Sec. 7, Art. 329 and 369; Kapp, pp. 390 to 423; Thomälen, Chap. 14; Lamb, Chap. 22; Franklin and Esty, Chap. 7; Steinmetz' "Elements," pp. 151 to 165; Steinmetz' "A.C. Phenomena," Chap. 23; Thompson's "Dynamos," Vol. 2, Chap. 11; Karapetoff, pp. 492 to 499; *Can. Elec. News,* August 5, 1905, A. L. Mudge, Operation of alternators in parallel; *Trans. Am. Inst. Elec. Eng.,* June, 1906, Morgan Brooks and M. K. Akers, The self synchronizing of alternators; *Elektrotech. Zeitschr.,* July 5, 1906, G. Benischke, Notes on the parallel running of alternators; *Elect'n Lond.,* September 18, 1903, B. Hopkinson; *Am. Elect'n,* October, 1902, F. P. Woodbury; *Eng. Rec.,* June 14, 1902, H. E. Longwell, The paralleling of alternators; *Elec. Wld. and Eng.,* June 14, 1902, J. M. Roman, Some notes on synchronizing; *Jour. Fr. Inst.,* April, 1902, Paul M. Lincoln; *Trans. Am. Inst. Elec. Eng.,* March, 1902, W. L. R. Emmet, Parallel operation of engine driven alternators; *Trans. Am. Inst. Elec. Eng.,* October, 1901, E. J. Berg, Parallel running of alternators; *Am. Elect'n,* September, 1901, A. S. McAllister, Parallel operation of alternators; *Elec. Rev. Lond.,* May 31, 1901, Alex. Russell, Notes on the theory of synchronizing motors and generators in parallel.

Object. To study the behavior of two alternators when operated in parallel.

Theory and Method. If two alternators having the same wave form are run with their electromotive force waves in unison and are generating the same voltage, they may be made to operate in parallel and divide the load in proportion to their ratings. It is found that the ideal conditions are seldom fulfilled. However, provided the difference in speed, voltage and phase of the two machines is not excessive, they may be made to fall into step and operate together satisfactorily. It is probable, however, that more or less synchronizing current will flow between the machines. This current will vary according to the relative excitation of the two machines. By synchronizing current is meant the current flowing between the two machines which, by its magnetizing or demagnetizing action, causes the terminal voltages of the machines to have the same value. This current is due to a resultant voltage between the two machines. It is practically wattless, but causes a small loss and consequent heating in the machine armatures.

Method of Synchronizing. When operating with low voltage machines the following method may be used. Referring to

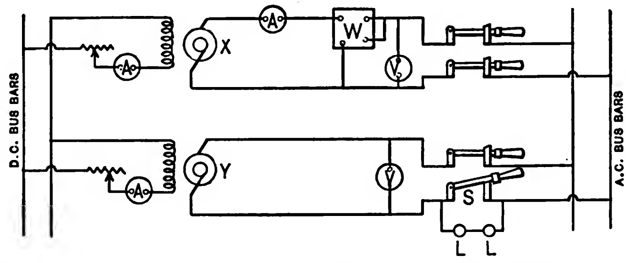

FIG. 65*A*. Connections for synchronizing two low voltage alternators.

Figure 65*A*, the alternators are excited to give normal pressure on both machines. All the machine switches, except switch *S*, are closed. A number of lamps, of a combined voltage equal to twice the voltage of one machine, are connected in series across switch *S*. As the alternators approach synchronism, a series of beats or

flashes occur in these lamps. When these beats have become sufficiently slow, switch S should be closed in the middle of a dark period. If the machines have been properly synchronized, and are of suitable design for parallel operation, no appreciable current will flow between them. If the switch is closed too soon, or too late, an instantaneous power current will flow. This will speed up the slower machine and slow down the faster machine, until they reach a state of equilibrium.

Ordinarily, alternators have a high armature pressure and lamps are not placed directly across the switch as shown in the diagram. In this case the lamps are placed in series across the secondaries of two transformers, the primaries of which are placed across the terminals of the two armatures. The transformers may be connected so that the alternators are in phase when the lamps are bright, or when they are dark. It is easier to note a change from maximum brilliancy than to determine a small change in speed when the lamps are dark. Custom varies with different engineers as to the method to be used in a given case.

Connections may also be made as in Figure 65B, where an

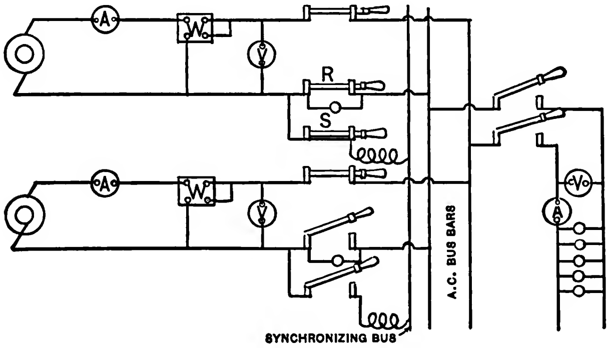

Fig. 65B. Connections for synchronizing two low voltage alternators using a synchronizing coil.

inductance without an iron core is connected across switch S in parallel with the lamps. When the machines are nearly in syn-

chronism, switch R is closed. When the lights have ceased to beat, switch S is closed, and the machines are in synchronism. By properly choosing this inductance it will be found to hold the machines in synchronism with quite a wide variation in frequency. This is especially important where the load fluctuates rapidly on one of the machines.

The value of this inductance should be so calculated that not more than full load current will flow in the worst position of the alternator electromotive force vectors; that is, an inductance sufficient to limit the current to full load, or less, with twice the voltage of one of the machines. In this calculation the value of the alternator armature inductance may be omitted. The size of wire to use in such a coil may be rather small in comparison with the maximum current it is to carry, as this current flows only momentarily. The use of the coil gives the operator more time to perform the necessary operations in synchronizing and at the same time insures against damage resulting from poor or accidental operation. It will be found, if several machines are to be connected in this manner, that the installation of a common synchronizing bus, as shown in Figure 65*B*, will aid in simplifying the connections in this case. By a further modification of the switching circuits, one coil may be used for several machines.

Data. In performing this experiment the following order of tests will be found to illustrate the principles of synchronizing.

1. Connect the machines as in Figure 65*A*. Adjust the machine pressures to the same value. Synchronize and connect in parallel. Read the synchronizing current. Also notice whether there is any momentary rush of current on closing the switch S, and its amount. In this case neither machine should be connected to an external load.

2. Increase the voltage of one of the machines above normal and decrease that of the other by the same amount, and proceed as under (1). Take a series of observations in this manner, varying the pressure by 100 percent., if possible. Care should be

taken that the armature current does not become excessive. The value of the synchronizing current should be read in each case.

No. 66. PARALLEL OPERATION OF ALTERNATORS. (LOAD DIVISION.)

References. Russell, Vol. 2, Chap. 7; Arnold, Vol. 4, Chap. 18 and 19; Standard Handbook, Sec. 7, Art. 329 to 369; Kapp, pp. 390 to 423; Thomälen, Chap. 14; Lamb, Chap. 22; Franklin and Esty, Chap. 7; Steinmetz' "Elements," pp. 151 to 165; Steinmetz' "A.C. Phenomena," Chap. 23; Thompson's "Dynamos," Vol. 2, Chap. 11; Karapetoff, pp. 492 to 499; *Can. Elec. News,* August, 1905, A. L. Mudge, Operation of alternators in parallel; *Trans. Am. Inst. Elec. Eng.,* June, 1906, Morgan Brooks and M. K. Akers, The self synchronizing of alternators; *Elektrotech. Zeitschr.,* July 5, 1906, G. Benischke, Notes on the parallel running of alternators; *Elect'n Lond.,* September 18, 1903, B. Hopkinson; *Am. Elect'n,* October, 1902, F. P. Woodbury; *Eng. Rec.,* June 14, 1902, H. E. Longwell, The paralleling of alternators; *Elec. Wld. and Eng.,* June 12, 1902, J. M. Roman, Some notes on synchronizing; *Jour. Fr. Inst.,* April, 1902, Paul M. Lincoln; *Trans. Am. Inst. Elec. Eng.,* March, 1902, W. L. R. Emmet, Parallel operation of engine driven alternators; *Trans. Am. Inst. Elec. Eng.,* October, 1901, E. J. Berg, Parallel running of alternators; *Am. Elect'n,* September, 1901, A. S. McAllister, Parallel operation of alternators; *Elec. Rev. Lond.,* May 31, 1901, Alex. Russell, Notes on the theory of synchronizing motors and alternators in parallel.

Object. To study the principles governing the division of load between two or more alternators.

Theory and Method. In Experiment 65, methods were studied for placing two or more alternators in parallel. These methods of synchronizing should be understood before attempting this experiment.

In the parallel operation of direct current machines it was learned that the load could be divided between two or more machines by varying the relative excitation of the machines. That is, the electromotive force necessary to send current through the armature of any machine was the numerical difference between the electromotive force generated by the machine, and the bus bar voltage. This difference in electromotive force, divided by the armature resistance, gave the value of the current furnished by that machine. As more load was placed on a given machine, more torque was required from its prime mover. This was brought about automatically. In direct current machine operation the speeds of two machines might be quite different and the machines still operate in parallel and divide the load.

In Experiment 65 it was found that a difference in voltage between two alternating current generators only served to cause a wattless current to flow between the two machines. This current is determined by the vector difference between the electromotive forces divided by the impedance of the armatures. This current served to magnetize the magnetic circuit of one of the machines and to demagnetize the magnetic circuit of the other machine, so as to produce the same terminal voltage on both machines. In fact, the excitation could be entirely removed from one of the machines and it would continue to operate, taking a heavy magnetizing current through its armature circuit.

In order to make the incoming machine take its share of the load after synchronizing, it is necessary to take a power component of current from its armature. This must be accomplished by increasing the driving torque on the incoming machine. In the case of an alternator driven by a direct current motor, this may be done by decreasing the field excitation of the motor. This tends to increase the speed of the set. The alternator must, of course, run in synchronism with the others with which it is in parallel. This tendency to increase speed causes the electromotive force vector to take a position such that a larger compo-

nent of the armature current is in phase with the generated voltage.

If the driving torque is decreased, the armature current flows in the opposite direction with respect to the terminal voltage and the machine becomes a motor running in synchronism with the system to which it is connected. The difference between the motor and the generator is, then, the direction of the driving torque.

If the field current be varied when the alternator is carrying a load, there must be a wattless current flowing in addition to the load current. In order to keep the armature losses at a minimum for any load, the field current must then be adjusted so that the electromotive force and current are in phase with each other. The same is true in the case of the synchronous motor, as will be seen in a later experiment.

Data. Load one of the machines with a non-inductive load. Synchronize the other machine with this one, as in Experiment 65. Connections should be made as in Figure 66.

1. Note the current and power supplied by the other machine

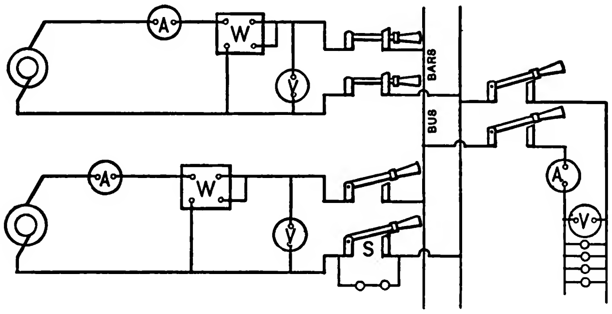

Fig. 66. Connections for determining the load division of alternators.

after synchronizing. Increase the field excitation of this machine, holding the terminal voltage of the other machine constant by adjusting its field excitation, if necessary. Note the effect of any relative change in the field excitations, on the operation of both machines.

2. With the fields adjusted to normal excitation and a non-inductive load on one of the machines, synchronize the other machine as before. Increase the driving torque of the second machine and note the effect on the operation of both machines.

3. With the same adjustment as to field excitation as in (2), decrease the driving torque of the second machine. If the machine is driven by a motor, disconnect this motor from its source of power. Note the effect on the amount of power furnished by each of the machines.

No. 67. OPERATION OF A SINGLE PHASE SYNCHRONOUS MOTOR UNDER VARIABLE LOAD AND CONSTANT FIELD CURRENT.

References. Standard Handbook, Sec. 8, Art. 76 to 97; Thomälen, Chap. 15; Russell, Vol. 2, Chap. 4; Karapetoff, pp. 500 to 504; Thompson's "Dynamos," Vol. 2, Chap. 10; Steinmetz' "Elements," pp. 141 to 152; Steinmetz' "A.C. Phenomena," Chap. 24 and 25; Franklin and Esty, Chap. 8; Lamb, Chap. 23; Arnold, Vol. 4, Chap. 13; *Elec. Wld.*, October 19, 1907; *Harvard Engng. Jour.*, January, 1908; April, 1908; January, 1909; C. A. Adams, The synchronous motor; *Trans. Am. Inst. Elec. Eng.*, June, 1902, C. P. Steinmetz' Notes on the theory of the synchronous motor; *Elec. Wld. and Eng.*, May 17, 1902, F. G. Baum, Synchronous motor calculations; *Elektrotech. Zeitschr.*, February 12, 1903, E. Rosenberg, An analysis of the no load current of synchronous motors.

Object. Although single phase synchronous motors have no wide commercial application, a study of their action becomes instructive in understanding the action of polyphase synchronous motors under the same conditions of operation.

Theory and Method. From Experiments 65 and 66 on the synchronizing of alternators, it is seen that the driving force can be removed from one of the machines, after it had been syn-

chronized with the other machine or with an alternating current line, and that it will then run as a motor. Such a motor will run absolutely at synchronous speed until the load is sufficient to pull it out of step. In case the motor falls out of step, it will come quickly to rest, even though the load is entirely removed. The disadvantage of this type of motor is that it is not self-starting. An advantage is that the machine maintains exactly synchronous

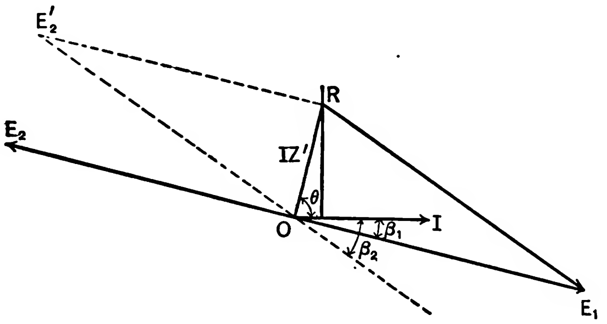

FIG. 67*A*. Clock diagram for a synchronous motor.

speed for all loads which it is capable of carrying and that it can be used to regulate the power factor of the system to which it is connected.

The methods of synchronizing are the same as those applied in the parallel operation of alternators and described in Experiment 65. In fact the operation of the synchronous motor may be considered as a special case of the parallel operation of alternators, in which, after the second machine has been brought up to speed and synchronized, the driving force has been changed from a positive to a negative value.

The simplest form of synchronous motor and driving generator would be obtained in the case of two single coil armatures, revolving in uniform fixed fields and connected in series with each other. It is evident that the motor is not self-starting. Supply each armature with a two-part commutator and the motor immediately becomes self-starting, and by a proper regulation of its field may be brought to exact synchronism with the generator.

A little consideration will show that when this condition prevails and the machines are commutating at the same instant, the act of commutation is unnecessary and the commutators may be replaced by slip rings, the result being a single phase synchronous motor, operated by a single phase alternator.

Suppose the counter pressure is equal to and in exact opposition to the impressed pressure at the moment the machines are synchronized. This relation is shown in Figure 67*A*, where OE_1 and OE_2 represent the impressed and the motor pressures, respectively. In this case no current can flow, and the result is that the motor begins to slow down, its pressure lagging behind the generator pressure, until its pressure vector assumes a new position, shown by the dotted line OE_2'. Since the two pressures are not in exact opposition, there is a resultant pressure OR and a current OI will flow, which lags behind OR by an angle determined by the resistance and synchronous reactance of the armature. The phase angle of the current with respect to the impressed pressure, OE_1, is β_1 and that with respect to the counter pressure, OE_2, is β_2.

Assuming sine waves, the power supplied to the motor is

$$P_1 = E_1 I \cos \beta_1. \qquad (67a)$$

Similarly, the power converted into mechanical torque, including the hysteresis, eddy current, friction and windage losses, is

$$P_2 = E_2 I \cos \beta_2. \qquad (67b)$$

The counter pressure of the motor, if running without load, will take such a position that Equation (67*b*) will equal the rotational losses in the motor. If the machine is loaded, the angle must be increased to correspond to the amount of power taken by the load. As the load becomes greater and greater, the lag becomes more and more, and a load will finally be reached beyond which the mechanical power begins to decrease. If this point be passed, the motor immediately falls out of step and comes to rest.

A useful study of the behavior of the synchronous motor may be made by the aid of the following tests and diagrams. The

synchronous impedance and the armature resistance should be measured carefully. A horizontal line should then be chosen for the direction of the current, as shown in Figure 67*B*.

Lay off a line OD at an angle θ determined by the resistance and synchronous impedance. Construct an arc about O with a radius equal to the impressed electromotive force, to scale. Choose a position of OE_1, as OA, making the angle β_1 with I as determined by the power factor of the load. On OD lay off a length equal to the synchronous impedance multiplied by the current I, as OD. Connect D and A and complete the parallelogram. If this construction is followed for each load, the other quantities may be determined in each case.

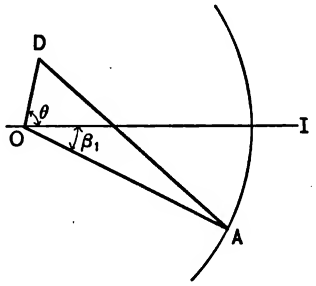

FIG. 67*B*. Clock diagram for a synchronous motor with constant impressed electromotive force.

Data. Make connections as in Experiments 65 and 66. Bring the motor up to speed by means of a direct current motor, adjust the pressure equal to the generator pressure, and synchronize, operating the direct current motor as a generator. Maintaining the impressed pressure, frequency and motor field constant at normal values, take a series of observations of armature current and power supplied to the motor, for various loads up to a load at which synchronism is broken. In a similar manner, take a series of observations for values of field current above and below normal.

Diagrams. Plot a series of clock diagrams similar to those described.

Questions. How can a synchronous motor be used to improve the power factor on a system having a large lagging load? Enumerate the sources of speed variation from the governor of the prime mover to the motor shaft of a direct current system having a transmission line and shunt motors and an alternating current system with a transmission line and synchronous motors.

No. 68. PHASE CHARACTERISTICS, OR V-CURVES, OF A SYNCHRONOUS MOTOR.

References. Standard Handbook, Sec. 8, Art. 76 to 97; Thomälen, Chap. 15; Russell, Vol. 2, Chap. 4; Karapetoff, pp. 500 to 504; Thompson's "Dynamos," Vol. 2, Chap. 10; Steinmetz' "Elements," pp. 141 to 152; Steinmetz' "A.C. Phenomena," Chap. 24 and 25; Franklin and Esty, Chap. 8; Lamb, Chap. 23; Arnold, Vol. 4, Chap. 15; *Elec. Wld.,* October 19, 1907; *Harvard Engng. Jour.,* January and April, 1908, and January, 1909, C. A. Adams, The synchronous motor; *Trans. Am. Inst. Elec. Eng.,* June 18, 1902, C. P. Steinmetz, Notes on the theory of the synchronous motor; *Elec. Wld. and Eng.,* May 17, 1902, F. G. Baum, Synchronous motor calculations; *Elektrotech. Zeitschr.,* February 12, 1903, E. Rosenberg, An analysis of the no load current of synchronous motors.

Object. To operate a synchronous motor at constant load and to determine the variation of the armature current and power factor with variations in field current.

Theory and Method. The armature current of a synchronous motor depends not only upon the load but also upon the counter pressure developed in the armature and its phase relation with respect to the impressed pressure. It is desired to investigate the manner in which a variation of the counter-pressure affects the armature current.

In the Introduction to Experiments on Synchronous Machines and in Experiment 67, it is shown that, in the case of a single phase synchronous motor, the intake is

$$P_1 = E_1 I \cos \beta_1,$$

and that the power converted mechanically, including all of the losses except the I^2R losses in the armature and fields, is

$$P_2 = E_2 I \cos \beta_2.$$

It is also shown that the armature current might lead, be in phase with, or lag behind the terminal electromotive force E_1,

depending upon the field excitation for a given load and impressed pressure. Assume the condition of constant impressed pressure and constant load, and that the hysteresis and eddy current losses remain constant. Since the speed is constant, the friction and windage loss will also be constant. The only variable part of the power supplied is the armature I^2R loss, and this will be a minimum when the armature current is a minimum. The total power supplied to the motor must then be least when the armature current is least for a given load, which is the case when the current is in phase with the terminal electromotive force. There will be a certain value of field current to give this minimum value of armature current. Any field current either greater or less than this will give a value of armature current increased graphically by the amount of the component necessary to furnish the proper armature reaction. A curve may be plotted for each load, between the value of the field current and the corresponding armature current. These curves are known as the phase characteristics, or *V*-curves, of the machine.

Clock diagrams may also be drawn for the machine, as in Experiment 67. These diagrams will show how a change in field current affects the phase relation of the impressed and counter pressure with respect to the current.

Data. Adjust the impressed pressure and motor load to the desired values, and maintain these constant. Take a series of observations of armature current, field current and intake of the motor, varying the field excitation from a low value to a value considerably above normal. Take similar sets of readings for no load, full load and intermediate loads; also for an over-load, if possible. For the no load curve the points should be chosen close together, particularly near the point of minimum current. Measure the resistance and synchronous reactance of the armature.

Calculate. The power factor for the various field excitations.

Curves. Plot curves taking as ordinates the armature current and as abscissas the corresponding field current. Plot curves showing the variation of the power factor with the field current

for each load. Plot a curve through the points of minimum current of each of the sets of curves.

Questions. What are some of the considerations which would lead you to recommend a synchronous motor for a given service? What method would you use for starting the motor for different classes of service? Show by diagrams how the synchronous motor may regulate the voltage at the receiving end of a transmission line?

Problem. A 1,000 kilovolt-ampere alternator is carrying 1,000 kilovolt-amperes at 70 percent. power factor. How large a synchronous motor is needed to raise its power factor to 80 percent.; to 90 percent.; to 100 percent.? How much may the kilowatt output of the machine be increased in each case? Compare the size of the synchronous motor to raise the power factor of this alternator to one which will also carry a load equal to half its rated output.

No. 69. CIRCLE DIAGRAM FOR A SYNCHRONOUS MOTOR.

References. Standard Handbook, Sec. 8, Art. 84 to 91; Arnold, Vol. 4, Chap. 15 and 16; *Elec. Eng. Lond.,* July 31, 1908, M. F. Creedy, The circle diagram of the synchronous motor; *Trans. Am. Inst. Elec. Eng.,* June, 1907, Morgan Brooks, Interaction of synchronous motors; *Harvard Engng. Jour.,* April, 1908, and January, 1909, *Elec. Wld.,* August 24, 1907, A. S. McAllister, Circular current loci of the synchronous motor.

Object. To construct a circle diagram for a synchronous motor and to determine the operation characteristics of the machine from the diagram.

Theory and Method. It is not always convenient to test large synchronous motors to full load, because of the consequent loss of power or on account of the lack of generator capacity. It is desirable to be able to predict the behavior of the machine from the no load tests.

As was explained in the Introduction to Experiments on Synchronous Machines, a circle diagram, originated by Blondel, may be constructed from the no load tests on the machine. The data necessary are the open circuit saturation curve, the values of the armature resistance and synchronous reactance, and the no load losses. These may be determined as in Experiments 49, 50 and 61. After determining these data for the machine, the resistance and reactance of a transmission line may also be included in their proper relations.

Consider the diagram, shown in Figure 69, constructed for constant terminal voltage. In this case only the machine con-

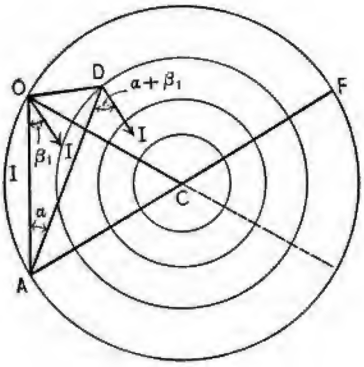

Fig. 69. Circle diagram for a synchronous motor.

stants will be considered. Choose a length OA, to scale, equal to the terminal voltage. On OA erect the isosceles triangle OCA with base angles equal to θ, this angle being determined from the relation

$$\cos\theta = \frac{R}{Z'},$$

where R is the armature resistance and Z' its synchronous impedance. With C as a center, circumscribe the circle AOF. The rest of the diagram is constructed according to the condition to be studied.

Suppose that the variation of power factor and current with

variation of field current is desired, motor output being kept constant, as in Experiment 68. Circles about C represent power output plus the no-load losses. The value of the current is determined by dividing the length OD by the synchronous impedance Z'. At unity power factor, D will be on the line OC and the value of I will be

$$I = \frac{\text{Output} + \text{losses}}{\text{Terminal voltage}}.$$

The point D on OC may then be found by the relation

$$OD = IZ'.$$

A circle through this point with center at C will be the locus of constant power output. From the open circuit saturation curve the value of the induced electromotive force corresponding to a given field current may be found. These values determine the length of AD and, consequently, the triangle AOD. From this the angle β_1, the angle $\alpha + \beta_1$, the power factor and the armature current may be determined, the latter quantity being

$$I = \frac{OD}{Z'}.$$

From these values the curves of Experiment 68 may be constructed.

Constant field current is represented by a constant value of AD. This in turn fixes the locus of D as the arc of a circle with center at A and radius equal to AD. From this the other quantities measured in Experiment 67 may be determined. Constant armature current is represented by OD constant, a locus being the arc of a circle about O and radius equal to IZ'. Other relations will also suggest themselves.

A more exact diagram may be constructed by using field currents instead of electromotive forces. In this construction the value of the field current corresponding to the terminal electromotive force E_1 is obtained from the open circuit characteristic and is taken as the base line OA of the diagram. OD is the field current corresponding to a given armature current, obtained

from the curve between armature current and field current on short circuit. *AD* is the field current for the condition chosen. The other constructions are then similar to those of the corresponding electromotive force diagram. The advantage of this form of diagram is that it takes into account the effect of armature reaction.

For the ordinary machine, the point *C* is not easily found on a diagram of convenient size or scale as θ (the angle between *OC* and *OI*) is nearly 90 degrees. With a transmission line taken into consideration, the complete circle may usually be constructed because of the smaller value of θ. The effect of a change in the value of θ, on the operation of a synchronous motor, may also be studied by this form of diagram.

Data. Secure data for the no-load saturation curve, the armature resistance and the synchronous reactance and also for the no-load losses of the machine.

Construct. A circle diagram for the machine, according to one of the methods described. Obtain values from this for the phase characteristics of the machine.

Curves. Plot curves taking as ordinates the armature current and as abscissas the corresponding field current. Plot curves showing the variation of the power factor with the field current for each load. Plot a curve through the points of minimum current of each of the sets of curves.

No. 70. DETERMINATION OF WAVE FORM BY AN OSCILLOGRAPH.

References. Karapetoff, pp. 620 to 626; Handbuch der Elek'tek, Vol. 2^4, pp. 108 to 119; Arnold, Vol. 1, pp. 192 to 196; Franklin and Esty, pp. 36 to 37 and 39 to 82; Thompson's "Dynamos," Vol. 2, pp. 237 and 238; *Elec. Wld. and Eng.*, May 6, 1905, F. A. Laws, A convenient form of oscillograph; May 2, 1903, Henry Hale, The Blondell oscillograph; *Elec. Jour.*, October, 1905, Rob Rankin, The cathode ray oscillograph; *Elec. Rev.*,

January 2, 1904, F. C. Perkins, Photographing alternating current waves with the new Duddell oscillograph; *Zeitschr. f. Elektrotech.*, July 16 and 23, 1905, W. Hornauer, The Siemens-Halske oscillograph; *Elec. Eng. Lond.*, July 25, 1902, Rud. Goldschmidt, An instrument for determining alternating current curves; *Trans. Am. Inst. Elec. Eng.*, June 20, 1902, R. E. Owens, A new curve tracing instrument; *Sib. Jour. Engng.*, April, 1903, G. S. Macomber, Cathode tube wave indicator; *Trans. Am. Inst. Elec. Eng.*, October, 1903, H. J. Ryan, The cathode ray alternating wave indicator; *Elektrotech. Zeitschr.*, November 5, 1903, J. Donitz, The wave measurer and its applications.

Object. To determine wave forms of current and pressure by means of an oscillograph.

Theory and Method. In using any method for determining wave form which requires a period of time spreading over several cycles, only average values can be determined and plotted. These values consist of average points on successive waves. The whole are then plotted as values for a single average wave. It is desirable, in phenomena having a transient term, to be able to plot the entire wave for several consecutive cycles.

Gerard first suggested a method of accomplishing this result for an alternator. In this method the alternator field is excited in the usual manner and the rotor is rotated at a very slow speed, the armature terminals being connected to the terminals of a D'Arsonval galvanometer coil. If the natural period of the galvanometer is high in comparison with the frequency of the pressure supplied by the alternator, the deflection of the galvanometer at each instant will be proportional to the instantaneous pressure. If the galvanometer mirror is made to reflect a beam of light upon a sheet of sensitized paper, and the paper is moved at a uniform rate transversely to the motion of the beam of light, the curve of potential will be permanently recorded in rectangular coördinates on the sensitized paper. The disadvantage of this method is that the machine cannot be run under normal conditions.

The oscillograph, first suggested by Blondel, is an adaptation

of this method. It consists essentially of a single loop or turn of light suspension wire or ribbon situated in the strong field of a permanent magnet or an electro-magnet. Upon this loop is mounted a very light mirror. The portion of the loop carrying the mirror is supported between bridges fairly close together and placed in the field. This arrangement gives a system having a very high natural period of vibration and it should respond to 3,000 or more cycles per second. The instrument is damped against its own natural vibrations by having the suspension partially immersed in a damping fluid of the proper viscosity.

The record may be made by throwing a reflected beam of light from the oscillograph mirror upon a synchronously oscillating mirror which reflects it upon a stationary screen where it may be viewed or traced, or it may be thrown upon a uniformly moving photographic film or plate. By this method, many phenomena of periodic and transient nature, as well as alternating waves, may be studied. By means of several moving systems throwing their beams upon the same film, the relation of one phenomenon to another may be studied. The instrument is, of necessity, delicate and requires a calibration for each new condition of use. This is accomplished by means of a direct current of known value. As each different make of instrument has its peculiarities of adjustment or calibration, no attempt is made here to cover this by directions. These are furnished by the manufacturer with each instrument.

The pressure wave is taken by connecting a non-inductive resistance in series with one of the vibrators, across the potential

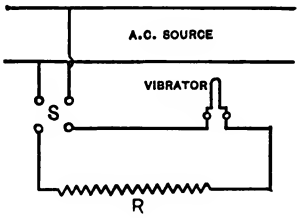

FIG. 70*A*. Oscillograph connections for determination of an electromotive force wave.

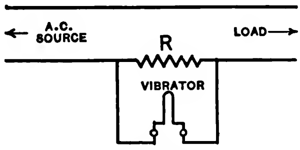

FIG. 70*B*. Oscillograph connections for determination of a current wave.

source, or across some value of the pressure, as shown in Figure 70*A*. The resistance *R* must be high enough to limit the current through the vibrator to a safe value.

Current curves are taken by shunting the vibrator across a low resistance in series with the load, as shown in Figure 70*B*. If current and pressure waves are taken at the same time they will be shown in their proper phase relation.

Data. Make a study of some form of oscillograph. Describe its parts and method of adjustment and calibration. Take, if possible, curves of pressure and current in non-inductive, inductive and capacity circuits, by tracing or on a photographic film. Discuss any peculiarities in the shape of these curves. Be sure to make a calibration for each curve taken.

No. 71. DETERMINATION OF WAVE FORM BY JOBERT'S METHOD.

References. Thompson's "Dynamos," Vol. 2, pp. 234 to 236; Handbuch der Elektrotech., Vol. 2^4, p. 96; Arnold, Vol. 1, pp. 190 to 192; Franklin and Esty, p. 35.

Object. To obtain the forms of pressure and current waves by means of a point by point method.

Theory and Method. Unless an oscillograph is available, it is usually necessary to obtain wave forms by means of a point by point method. The following method may be used for such a determination. A contact-maker is mounted on the shaft of the alternator supplying the energy or on the shaft of a synchronous motor operated from the alternating current source. The instantaneous pressure is measured, the connections being as shown in Figure 71. The condenser is charged to the instantaneous pressure from the source by means of a succession of impulses. The amount of the charge is shown by the equation

$$Q = CE,$$

where

$Q =$ the quantity of electricity,
$C =$ the capacity of the condenser,
$E =$ the instantaneous pressure.

The switch S is then thrown so that the condenser discharges through a ballistic galvanometer G, which should be calibrated to give the value of the electromotive force E in volts. By changing the location of the brush of the contact-maker through 360 electrical degrees, points for two half waves may be obtained. Pressure waves are taken by connecting as in Figure 71. Current waves may be obtained by connecting the contact-maker circuit in shunt across a resistance which is in series with the load. By means of a double throw switch, current and pressure waves may be obtained in their proper phase relation.

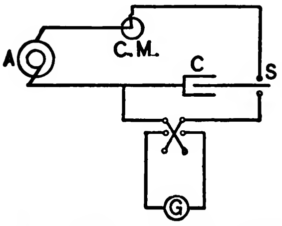

FIG. 71. Joubert's method of determining wave form.

Data. Obtain points for an electromotive force wave and for a current wave, by this method.

Curves. Plot the points obtained in rectangular coördinates, using angular positions, reduced to electrical degrees, as abscissas and electromotive force and current as ordinates. Multiply the instantaneous ordinates of the two waves together and plot the watt curve.

Question. If the contact-maker is driven by a synchronous motor which "hunts," how will the observations be affected?

No. 72. DETERMINATION OF WAVE FORM BY BEDELL'S METHOD.

Object. To determine the forms of pressure and current waves by means of a contact-maker.

Theory and Method. This method makes use of the same form of contact-maker as in Experiment 71 but differs in the method of measuring the instantaneous pressure. An electro-

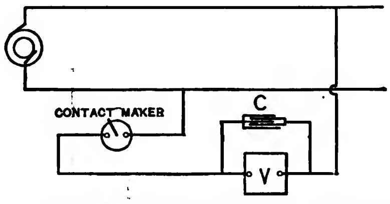

FIG. 72*A*. Bedell's method of determining wave form.

static voltmeter of the type designed by Lord Kelvin takes very little power and may be used directly to measure the instantaneous pressures if placed in the circuit with the contact-maker as shown in Figure 72*A*. It is advisable to shunt a condenser *C* around the voltmeter to make the instrument more nearly dead beat.

The electrostatic voltmeter does not indicate low values of potential accurately and, in fact, is not graduated for the lower part of the scale. To bring the needle into a readable part of

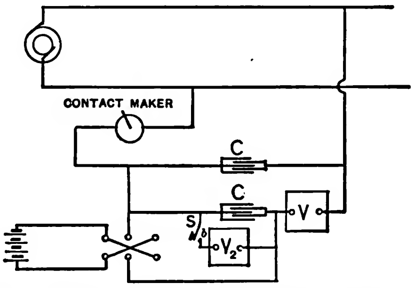

FIG. 72*B*. Bedell's method of determining wave form.

the scale, a second condenser C_1 is connected in the circuit as shown in Figure *72B*. The potential of this auxiliary condenser is determined by the voltmeter V_2. The electrostatic voltmeter should be placed in a position where it will not be affected by vibrations or outside electrostatic influences.

Connections. Bedell's method of connection is shown in Figure *72B*. Another method of connection which has given satisfactory results, is shown in Figure *72C*. Current curves

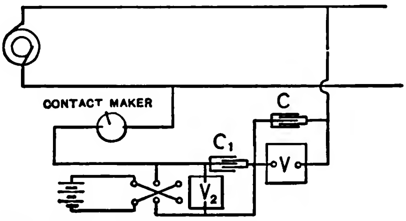

Fig. *72C*. Modification of Bedell's method of determining wave form.

can be obtained by taking the drop across a series resistance as the source of potential for the contact-maker circuit.

Data. Data for electromotive force and current curves should be obtained by this method, using some good form of contact-maker.

Curves. Plot the points obtained in rectangular coördinates, using angular positions, reduced to electrical degrees, as abscissas and electromotice force and current as ordinates. Multiply the instantaneous ordinates of the two waves together and plot the watt curve.

No. 73. DETERMINATION OF WAVE FORM BY MERSHON'S METHOD.

References. *Elec. Wld.*, Vol. 18, p. 140, R. D. Mershon; Karapetoff, pp. 616 to 618; Thompson's "Dynamos," Vol. 2, p. 235.

Object. To determine the form of pressure and current waves by means of a contact-maker.

Theory and Method. It is usually difficult to determine with a fair degree of accuracy the values of potential over the entire range needed in obtaining the points for an electromotive force or current curve. This method aims at obtaining the low values of electromotive force accurately. It consists, essentially, of a determination of the instantaneous values from the contact-maker by means of a potentiometer method. Instead of a galvanometer, a telephone receiver is used since the potential from the contact-maker is intermittent. The method of connection of the apparatus is shown in Figure 73. Instead of using a standard cell,

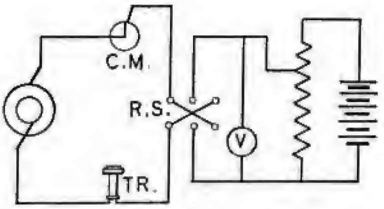

FIG. 73. Mershon's method of determining wave form.

as is usual with a potentiometer, use may be made of a calibrated direct current voltmeter, which gives the instantaneous values with sufficient accuracy.

Any sort of resistance, even a trough-type water rheostat with an intermediate movable plate, is suitable for the potentiometer. Two readings of the voltmeter should be taken for each point of the curve. Beginning with a loud sound in the telephone receiver, the potentiometer is adjusted for no sound. The adjustment is then repeated from a point of loud sound on the opposite side of the neutral position on the potentiometer. The average of the two voltmeter readings is the instantaneous pressure.

Data. Readings should be taken to determine potential and current curves by this method.

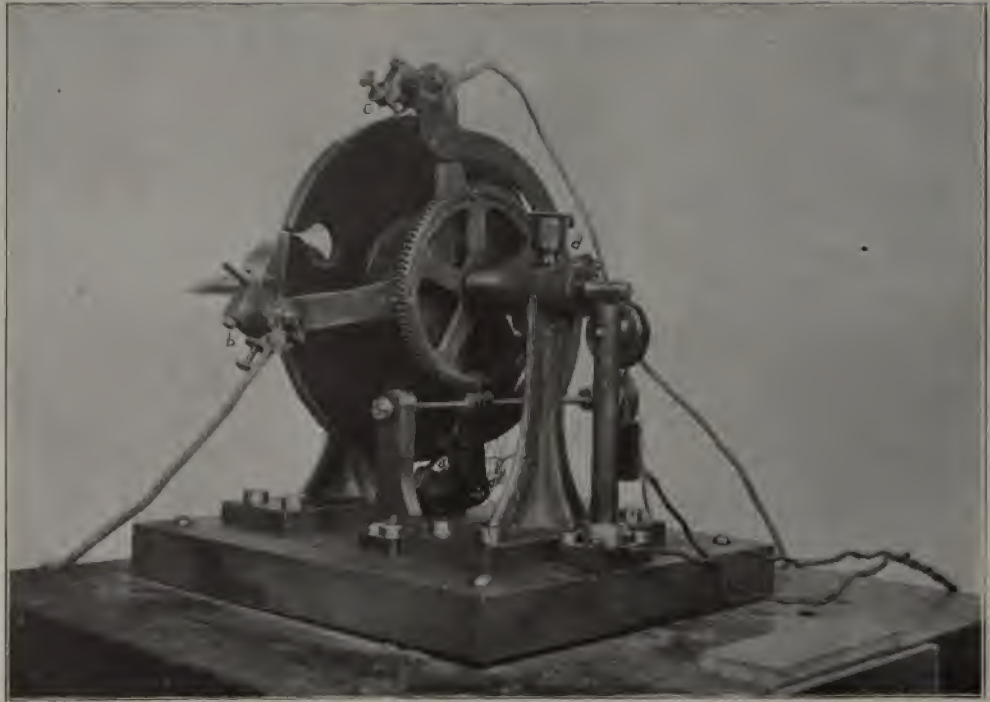

Fig. 74*A*. Rotating commutator with adjustable brushes for tracing magnetic waves.

No. 74. DETERMINATION OF WAVE FORM OF MAGNETIC FLUX.

References. *Bull. Bu. of Standards,* Vol. 4, No. 4, May, 1908, Lloyde and Fisher, An appliance for determining the wave form of magnetic flux; Thesis, Univ. of Ill., 1909, Farmer and Mathewson.

Object. To determine the wave form of magnetic flux in a given coil.

Theory and Method. In the determination of the losses in a a given iron core, it is convenient to know the wave form of the magnetic flux threading the iron. The following method is used by the United States Bureau of Standards. The only instrument necessary is a portable voltmeter of the D'Arsonval type in addition to the special form of contact-maker described in the Bulletin of the Bureau of Standards mentioned in the references. The following description of the contact-maker is quoted directly from the Bulletin and the accompanying cut, Fig. 74*A*, was also obtained from the same source.

"The essential part of the apparatus is a rotating commutator which reverses the contacts twice per cycle. An ebonite disk is mounted on a shaft between bearings; to its circumference is fastened a thin conducting strip with gaps at two points 180° apart. Four brushes, equally spaced, bear upon the metal rim of the rotating disk, and their joint action is that of commutation. A carrier, of brass, having four arms, is mounted loose upon the shaft; the arms support the brush holders. Screwed fast to the carrier is a gear wheel which meshes with a worm. A graduated circular scale is attached to the carrier, and is read by means of a fixed index fastened to the base.

"By means of the worm and gear the brushes may be set in any desired position. The brushes are thoroughly insulated from each other and from the carrier by means of ebonite strips at the extremities of the arms.

"The shaft of the rotating commutator may be coupled directly

to the generator shaft. The instrument was designed for use with a four-pole machine, running at 1800 revolutions per minute. Fig. 74*A* shows a photograph of the commutator in position for use. The principal dimensions are:

Circumference of disk	56 cm.
Thickness of metal rim	1.5 mm.
Width of gaps in rim	2 mm.
Gear diameter, 96 teeth	10.2 cm.

"The brushes *a c* are connected to a secondary coil wound around the flux to be measured. The brushes *b d* are connected to an indicating direct-current instrument, such as a d'Arsonval galvanometer or Weston voltmeter. Any instrument whose deflection is proportional to the first power of the current will answer the purpose.

"To plot a curve of magnetic induction, a reading on the Weston instrument is taken for a definite position of the brushes. Then successive readings are taken, the brushes being advanced the same number of degrees each time until they have been shifted 180°. In practice it suffices to shift the brushes only 90°, corresponding to a half cycle, since the readings repeat themselves in the second quadrant when only the odd harmonics of the fundamental frequency are present, and the apparatus is suited only to waves of this character. This condition means that the positive and negative lobes of the wave shall be similar, a condition which will be fulfilled if the magnetization is produced by a well-designed and well-constructed generator."

The theory of the instrument is as follows:

Let $\phi =$ the magnetic flux,
$f =$ the frequency,
$T = 1/f =$ the period,
$n =$ the turns in the exciting coil,
$E =$ the reading of the direct current voltmeter,
and $t =$ the time of commutation.

The intantaneous electromotive force induced in the coil by the flux ϕ is

$$e = -n\frac{d\phi}{dt},$$

or

$$\phi = -\frac{1}{n}\int edt.$$

Now $\int edt$ is the area of an electromotive force wave between the limits taken for the commutation. The contact-maker is made to make contact over 180 electrical degrees and reverses for the next 180 electrical degrees.

The change in flux is, then (Figure 74*B*),

$$\phi_{t+\frac{T}{2}} - \phi_t = -\frac{1}{n}\int_t^{t+\frac{T}{2}} edt.$$

If a periodic function is chosen in which the positive and nega-

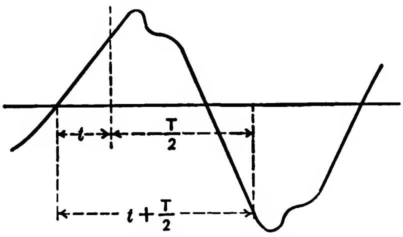

Fig. 74*B*. Integration of an electromotive force wave to obtain a flux wave.

tive lobes are similar, as is the case with most alternating current waves, then

$$\phi_{t+\frac{T}{2}} = -\phi_t.$$

But, since the direct current voltmeter shows the average of the electromotive force wave, its reading is

$$E = \frac{2}{T}\int_t^{t+\frac{T}{2}} edt.$$

That is, the reading is the area of the electromotive force wave divided by its base. Substituting this value in the previous equations, we have

$$\int_t^{t+\frac{T}{2}} e\,dt = \frac{ET}{2},$$

and

$$\phi_{t+\frac{T}{2}} - \phi_t = -2\phi_t,$$

then

$$\phi_t = \frac{ET}{4n}.$$

Hence, the reading of the direct current voltmeter is a measure of the instantaneous flux at the instant the contact-maker commutates. If the number of turns is not known, the wave form may still be plotted in percentage values. If the maximum value of ϕ is determined, the other values may be obtained by proportion.

Data. With a form of contact-maker as described in this experiment, determine one or more flux waves.

No. 75. DETERMINATION OF THE PRESSURE CURVES OF AN ALTERNATOR.

Object. To obtain the pressure curves of an alternator under no load and under load conditions.

Theory and Method. The pressure curve of an alternator varies in shape and position with respect to the field poles, according to the load in the external circuit. At no load, its shape depends upon the distribution of flux from the field poles and upon the angular position of the conductors on the armature. At no load, the maximum point of the electromotive force wave occurs when the armature conductors are under the centers of the field poles.

The energy component of the load current causes the trailing pole tip to become stronger than the leading pole tip and hence the flux distribution and electromotive force waves are changed in shape. The quadrature component of the load current causes ampere turns which directly oppose or aid the field ampere turns, thus weakening or strengthening the field flux but not distorting it.

If a contact-maker is attached to the shaft of the alternator, the relative position of the electromotive force wave in respect to the field poles may be determined, as well as the shape of the wave. One of the methods used in Experiments 70, 71, 72 and 73 may be used in this experiment.

Data. Determine the pressure curve of an alternator under non-inductive, inductive and capacity loads.

Curves. Plot curves in rectangular coördinates, using angular position of the contact-maker in electrical degrees as abscissas. Sketch in the position of the pole face from a determination of the position of its center line in respect to the contact-maker and the relative proportion of the armature covered by the pole face.

No. 76. DETERMINATION OF THE PRESSURE AND CURRENT CURVES OF AN ALTERNATING CURRENT ARC LAMP.

References. *Elec. Wld. and Eng.*, November 10, 1900, June 21, 1902, C. Wilder, The alternating current arc; Steinmetz' "Radiation," Lecture 8; Steinmetz' "A.C. Phenomena," pp. 578 to 585.

Object. To determine the potential and current curves of an alternating current arc lamp.

Theory and Method. The apparent resistance of an arc depends upon the amount of current. It is high for low currents and low for high currents, hence it varies through a cycle for

each alternation of current through the arc. An oscillograph, or a contact-maker, should be used to measure the waves.

Data. In order to study the effect of the arc on the wave shape, a sine wave of pressure should be used on the circuit. Take readings to determine the shape and phase angle of the potential and current waves. Measure the power consumed by the portion of the circuit over which the wave form is being taken.

Curves. Plot the current and pressure curves.

Calculations. From the current and electromotive force waves determine the power and check against the wattmeter readings.

Question. Explain how a circuit may have a power factor less than unity and still have the potential and current waves crossing the axis at the same point.

No. 77. DETERMINATION OF THE PRESSURE AND CURRENT CURVES OF A TRANSFORMER PRIMARY AND SECONDARY IN THEIR PROPER PHASE RELATIONS.

Object. To obtain a knowledge of what actually occurs in the working of a transformer under various conditions of operation.

Method. There is nothing new in the mode of procedure here. One of the methods of tracing curves described in Experiments 70, 71, 72 and 73, may be used. Figure 77 outlines a method of connection. R_1 is a non-inductive resistance for obtaining the primary pressure curve; the drop across R_1' is, at any instant, proportional to the instantaneous value of the primary current; the secondary pressure is taken directly from the secondary terminals, or, if this pressure is too high, a resistance R_2 may be used as shown in the diagram; and the secondary current curve is traced by measuring the drop across R_2'. The dotted lines indicate how connections may readily be made with the curve tracing instrument. This may be a contact maker with auxiliaries or, preferably, an oscillograph. If a contact maker is used, the points

on all four curves should be obtained for each setting of the contact; if an oscillograph is employed, care should be taken to get a record showing the curves in their proper phase relation.

Data. Obtain curves of primary pressure and current and of secondary pressure and current in their several phase relations for no load and for non-inductive, inductive and capacity loads.

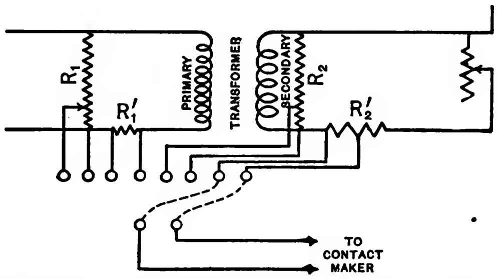

FIG. 77. Connections for obtaining the electromotive force and current waves of the primary and secondary of a transformer.

Curves. Plot the above mentioned curves and, in addition, plot curves of primary and secondary watts. These latter curves may be obtained by taking the products of instantaneous values of the pressure and current.

Discuss. The phase relations and the forms of these curves, giving full reasons for each peculiarity.

Question. How could power factor and efficiency be figured from these results?

No. 78. HARMONIC ANALYSIS OF WAVE FORM.

References. *Jour. Electricity, Power and Gas,* 1900, Vol. 9, p. 61, D. H. Fry, Harmonic analysis; Steinmetz' "A.C. Phenomena," pp. 585 to 596; Karapetoff, pp. 626 to 635; Ryan, pp. 42 to 59; Arnold, Vol. 1, p. 152; Thompson's "Dynamos," Vol. 2, pp. 64 to 72; Russell, Vol. 2, Chap. 3; *Elec. Wld. and Eng.,* March 25, 1905, G. R. Rowe, Harmonic analysis; *Elektrotech. u. Maschinenbau,* September 23, 1906, R. H. Haza, A new method of decomposing a periodic curve into its harmonics; *Stevens Ind.,* October,

1906, S. A. Hazeltine, Analysis of alternator waves; *Mech. Eng.*, May 13, 1905, C. F. Smith, Analysis of alternator waves; *Elec. Wld. and Eng.*, May 28, 1904, S. M. Kintner, Alternating current wave form analysis; *Elektrotech. Zeitschr.*, March 16, 1905, C.

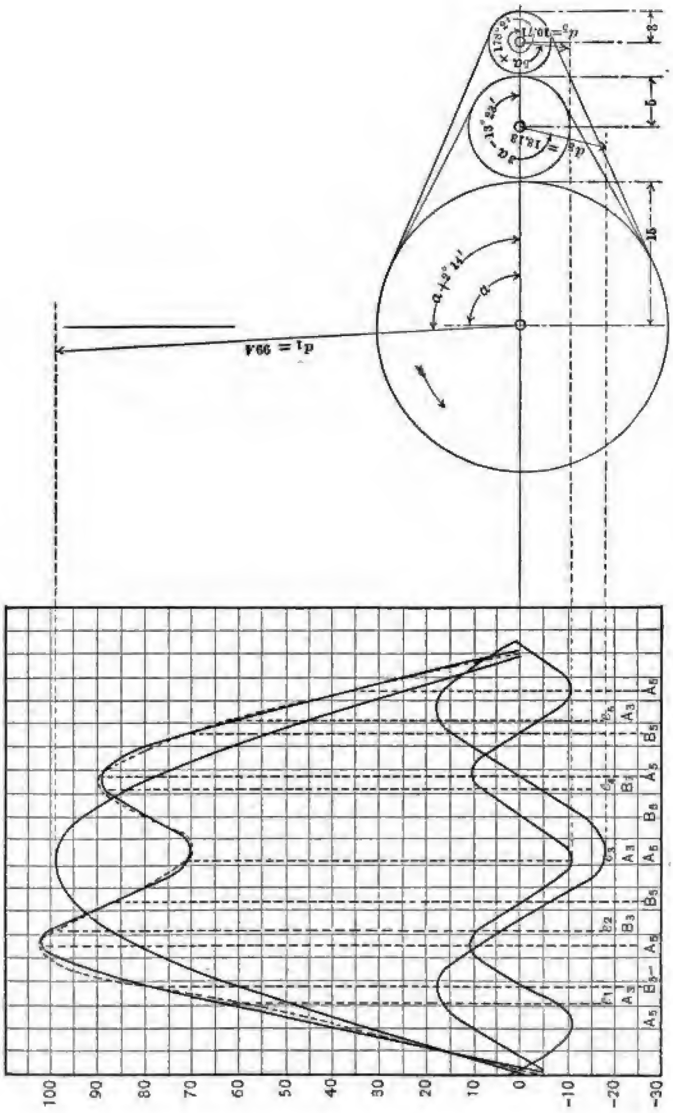

FIG. 78*A*. Analysis of a complex wave form showing the fundamental and the third and fifth harmonics.

FIG. 78*B*. Vectors representing the wave form shown in Fig. 78*A*.

Runge, Methods of analyzing sine waves; *Elect'n Lond.,* February 5, 1902, John Perry, Analysis of waves by algebraic reductia.

Object. The growing importance of a study of wave forms in reference to their effects on alternating current apparatus, makes it of value to know some simple method of separating an irregular wave into its several component harmonics.

Theory and Method. Any irregular single valued wave form may be considered as composed of a fundamental sine wave and a number of higher harmonics of this fundamental, the amplitude and phase position of each of these sine waves being of such values that the algebraic sum of their simultaneous ordinates is at any instant equal to the ordinate of the principal. All alternating current and pressure waves, found in commercial apparatus, are symmetrical with respect to time; *i. e.,* if the positive and negative loops are each divided into vertical strips by (n) equidistant ordinates, the ordinate $(n - a)$ of the positive loop will equal the ordinate $(n - a)$ of the negative loop. This condition can only obtain when the harmonics are all odd. These harmonics are not mathematical fictions; on the contrary, they have a real existence; and an investigation of their origin shows that only odd harmonics are generated.

A conductor revolving uniformly in a uniform magnetic field of constant value will have generated in it a sine wave of pressure. If the terminals of this conductor be connected to the terminals of a constant impedance, a sine wave of current will result.

Any variation from the sine wave, either in the pressure or the current wave of any circuit, will be due to one or more of the following causes:—

1. Non-uniform distribution of magnetic field.
2. Non-uniform velocity of conductor.
3. Non-uniform pulsation of the magnetic field.
4. Pulsation of magnetic permeability.
5. Pulsation of dielectric permeability.
6. Pulsation of resistance.

As a concrete example of analysis, take the complex pressure wave indicated in Figure 78*A*. This may be represented by an equation of the form

$$e = d_1 \sin(\alpha + \beta_1) + d_3 \sin(3\alpha + \beta_3) + d_5 \sin(5\alpha + \beta_5) + \cdots, \quad (78a)$$

where (e) is the value of the instantaneous pressure of the principal, and d_1, d_3 and d_5 are the amplitudes, respectively, of the fundamental, the third and the fifth harmonic. The phase differences between the harmonics and the principal are β_1, β_3 and β_5, each angle being expressed in terms of the harmonic with which it is associated.

Each component may be split up into two sine waves at 90° difference in phase; and the expression may be written

$$e = a_1 \sin \alpha + a_3 \sin 3\alpha + a_5 \sin 5\alpha + \cdots$$
$$+ b_1 \cos \alpha + b_3 \cos 3\alpha + b_5 \cos 5\alpha + \cdots, \quad (78b)$$

where

$$a_m^2 + b_m^2 = d_m^2,$$

(m) being a general subscript.

In this case, the sine components are all in phase with the principal; and each cosine component lags behind its corresponding sine wave by 90 degrees. If the base line of the principal is divided by vertical lines into $(n + 1)$ equal parts, there will be (n) ordinates representing instantaneous values of (e). Call these e_1, e_2, e_3, etc.; the abscissas of these ordinates will be

$$\frac{\pi}{n+1}, \frac{2\pi}{n+1}, \frac{3\pi}{n+1}, \text{ etc.},$$

for which $\dfrac{k\pi}{n+1}$ is a general expression.

The value of any ordinate e_k will be

$$e_k = a_1 \sin\left(\frac{k\pi}{n+1}\right) + a_3 \sin 3\left(\frac{k\pi}{n+1}\right) + a_5 \sin 5\left(\frac{k\pi}{n+1}\right) + \cdots$$
$$+ b_1 \cos\left(\frac{k\pi}{n+1}\right) + b_3 \cos 3\left(\frac{k\pi}{n+1}\right) + b_5 \cos 5\left(\frac{k\pi}{n+1}\right) + \cdots. \quad (78c)$$

Thus (n) simultaneous equations of the first degree are established, from which the coefficients a_m and b_m may be evaluated as follows:—

$$a_m=\frac{2}{n+1}\left[e_1\sin\left(\frac{m180}{n+1}\right)+e_2\sin\left(\frac{2m180}{n+1}\right)+\cdots+e_n\sin\left(\frac{nm180}{n+1}\right)\right],$$

$$b_m=\frac{2}{n+1}\left[e_1\cos\left(\frac{m180}{n+1}\right)+e_2\cos\left(\frac{2m180}{n+1}\right)+\cdots+e_n\cos\left(\frac{nm180}{n+1}\right)\right],$$

or

$$a_m=\frac{2}{n+1}\sum_{k=1}^{k=n} e\sin\left(\frac{mk\pi}{n+1}\right), \qquad (78d)$$

and

$$b_m=\frac{2}{n+1}\sum_{k=1}^{k=n} e\cos\left(\frac{mk\pi}{n+1}\right). \qquad (78e)$$

Having determined all the coefficients (a) and (b), the amplitudes d_m may be computed as previously indicated. The values of β_m are easily determined; for

$$\beta_m=\tan^{-1}\frac{b_m}{a_m}.$$

If b_m is negative, β is negative and the harmonic lags; if b_m is positive, β is a leading angle. If a_m is negative, β_m is greater than 90°; if a_m is positive, β_m is less than 90°.

Figure 78*A* shows the pressure wave of an alternator. The base line is divided into six equal parts; hence

$$n+1=6$$

and

$$\frac{\pi}{n+1}=30°.$$

$e_1=65$, $e_2=101.6$, $e_3=71$, $e_4=89$ and $e_5=59$.

$$a_1=\tfrac{1}{3}(65\sin 30°+101\ 6\sin 60°+71\sin 90°$$
$$+89\sin 120°+59\sin 150°)=+99.36.$$

$$b_1 = \tfrac{1}{3}(65 \cos 30° + 101.6 \cos 60° + 71 \cos 90° + 89 \cos 120° + 59 \cos 150°) = +3.77.$$

$$d_1 = \sqrt{a_1^2 + b_1^2} = \sqrt{(99.36)^2 + (3.77)^2} = 99.4.$$

$$\beta_1 = \tan^{-1}\frac{b_1}{a_1} = \tan^{-1}\frac{+3.77}{+99.36} = 2° \; 14' \text{ lead.}$$

$$a_3 = \tfrac{1}{3}(65 \sin 90° + 101.6 \sin 180° + 71 \sin 270° + 89 \sin 360° + 59 \sin 450°) = +17.66.$$

$$b_3 = \tfrac{1}{3}(65 \cos 90° + 101.6 \cos 180° + 71 \cos 270° + 89 \cos 360° + 59 \cos 450°) = -4.2.$$

$$d_3 = \sqrt{a_3^2 + b_3^2} = \sqrt{(+17.66)^2 + (-4.2)^2} = 18.13.$$

$$\beta_3 = \tan^{-1}\frac{b_3}{a_3} = \tan^{-1}\frac{-4.2}{+17.66} = 13° \; 23' \text{ lag.}$$

$$a_5 = \tfrac{1}{3}(65 \sin 150° + 101.6 \sin 300° + 71 \sin 450° + 89 \sin 600° + 59 \sin 750°) = -10.7.$$

$$b_5 = \tfrac{1}{3}(65 \cos 150° + 101.6 \cos 300° + 71 \cos 450° + 89 \cos 600° + 59 \cos 750°) = +0.366.$$

$$d_5 = \sqrt{a_5^2 + b_5^2} = \sqrt{(-10.7)^2 + (+0.366)^2} = 10.71.$$

$$\beta_5 = \tan^{-1}\frac{b_5}{a_5} = \tan^{-1}\frac{+0.366}{-10.7} = 178° \; 2' \text{ lead.}$$

All harmonics higher than the fifth were found negligible. The equation of the curve as analyzed is:—

$$e = 99.4 \sin(\alpha + 2° \; 14') + 18.13 \sin(3\alpha - 13° \; 23') + 10.71 \sin(5\alpha + 178° \; 2'),$$

and is actually represented by the dotted curve. The sine components are represented graphically in the diagram.

Figure 78*B* shows the radii vectores of the curve in their several phase relations. Each vector is represented as being rigidly connected to a pulley of radius inversely commensurate with its angular velocity. The pulleys for the higher harmonics are each belted to that of the fundamental. This system when rotated will generate by projection upon the proper ordinates, the component curves from which the original curve may be constructed by addition of ordinates. The instantaneous ordinates of the sine components are represented by the vertical projections of the vectors, in this case for $\alpha = 90°$; and the algebraic sum of these projections is the ordinate e_3 of the curve.

Accuracy. This method may be carried out with almost any degree of accuracy by choosing a large number of ordinates (n). The labor is, of course, proportionately increased. A greater error is likely to be made in determining a phase displacement than an amplitude.

Short Method. If, in Equation (78*e*) the number of divisions $(n + 1)$ be made twice the desired harmonic,

$$a_m = \frac{1}{m} \sum_{k=1}^{k=2m-1} e \sin\left(\frac{k\pi}{2}\right) \tag{78f}$$

and

$$b_m = \frac{1}{m} \sum_{k=1}^{k=2m-1} e \cos\left(\frac{k\pi}{2}\right). \tag{78g}$$

Expanding,

$$a_m = \frac{1}{m}\left(e_1 \sin\frac{\pi}{2} + e_2 \sin\frac{2\pi}{2} + e_3 \sin\frac{3\pi}{2} + \cdots\right)$$

$$= \frac{1}{m}(e_1 - e_3 + e_5 - e_7 + \cdots). \tag{78h}$$

Similarly,

$$b_m = \frac{1}{m}(-e_2 + e_4 - e_6 + e_8 - \cdots). \tag{78i}$$

Applying this to the case in hand, the (a) and (b) coordinates for the third harmonic (designated A_3 and B_3) are

A_3 65, 71, 59.

B_3 101.6, 89.

$a_3 = \frac{1}{3}(65 - 71 + 59) = + 17.66.$

$b_3 = \frac{1}{3}(- 101.6 + 89) = - 4.2.$

$d_3 = 18.13.$

$\beta_3 = 13^\circ\ 23'$ lag.

For the fifth harmonic,

A_5 36, 102.5, 71, 89.8, 36.6.

B_5 78.7, 89.2, 77.2, 72.77.

$a_5 = \frac{1}{5}(36 - 102.5 + 71 - 89.8 + 36.6) = - 9.74.$

$b_5 = \frac{1}{5}(- 78.7 + 89.2 - 77.2 + 72.77) = + 1.2.$

$d_5 = 9.8.$

$\beta_5 = 172^\circ\ 58'$ lead.

The equation is, therefore:—

$$e = 99.4 \sin (\alpha + 2^\circ\ 14') + 18.13(3\alpha - 13^\circ\ 23') + 9.8 \sin (5\alpha + 172^\circ\ 58'),$$

which compares favorably with the more exact method.

Accuracy. This method is limited by the accuracy obtained by dividing the base line of the wave into a number of parts equal to twice the harmonic desired. It fails for the fundamental. This may be obtained by the general method or by a number of readily suggested graphical methods.

Where the curve contains multiple harmonics (ninth, fifteenth, twenty-first, etc.) Equations (78h) and (78i) do not hold. The error may be eliminated, however, by finding those harmonics first, reconstructing the curve and proceeding as above. Fortu-

nately, it is seldom that harmonics higher than the seventh are of sufficient amplitude to be important. However, no thorough investigation should stop with the seventh.

Data. Taking any wave form obtained in the laboratory, preferably an irregular one, analyze it and construct diagrams similar to those in Figure 78*A*.

Questions. What are some of the effects of higher harmonics in commercial apparatus? In transmission lines? How does self-induction affect the amplitudes of the higher harmonics in a wave form? What effect has capacity? What are the effects of self-induction and capacity on the phase relation of the harmonics and the fundamental?

No. 79. ANALYSIS OF UNIVALENT WAVE FORM.

References. Thompson's "Dynamos," Vol. 2, Chap. 2; Russell, Vol. 2, Chap. 3; Arnold, Vol. 1, Chap. 9; *Trans. Am. Inst. Elec. Eng.*, May, 1910; see references to Experiment 78.

Object. To develop a method for analyzing any single valent periodic wave.

Theory and Method. Any univalent periodic function may be expressed by Fourier's series as

$$y = A_0 + A_1 \cos \beta + A_2 \cos 2\beta + A_3 \cos 3\beta + \cdots$$
$$+ B_1 \sin \beta + B_2 \sin 2\beta + B_3 \sin 3\beta + \cdots, \qquad (79a)$$

where y is a function of β. Or

$$y = A_0 + A_1 \cos \beta + B_1 \sin \beta + A_2 \cos 2\beta + B_2 \sin 2\beta + \cdots.$$

Let

$$A_1 = C_1 \cos \alpha_1,$$

and

$$B_1 = C_1 \sin \alpha_1,$$

where

$$C_1 = \sqrt{A_1^2 + B_1^2} \quad \text{and} \quad \tan \alpha_1 = \frac{B_1}{A_1}.$$

Equation (79*a*) rewritten becomes

$$y = A_0 + C_1 \cos \alpha_1 \cos \beta + C_1 \sin \alpha_1 \sin \beta + \cdots$$
$$= A_0 + C_1 \cos(\beta - \alpha_1) + C_2 \cos(2\beta - \alpha_2) + C_3 \cos(3\beta - \alpha_3) + \cdots,$$

a simple series.

To obtain the values of the coefficients and angles for a given wave, the following method may be used.

Multiply Equation (79*a*) by $d\beta$ and integrate over one cycle.

$$\int_0^{2\pi} y d\beta = A_0 \int_0^{2\pi} d\beta + A_1 \int_0^{2\pi} \cos \beta d\beta + A_2 \int_0^{2\pi} \cos 2\beta d\beta + \cdots$$
$$+ A_n \int_0^{2\pi} \cos n\beta d\beta + B_1 \int_0^{2\pi} \sin \beta d\beta + B_2 \int_0^{2\pi} \sin 2\beta d\beta + \cdots$$
$$+ B_n \int_0^{2\pi} \sin n\beta d\beta. \qquad (79b)$$

Taking any of these terms,

$$A_n \int_0^{2\pi} \cos n\beta d\beta = \frac{A_n}{n} \Big| \sin n\beta \Big|_0^{2\pi}.$$

But

$$\sin 2\pi n = 0,$$

and

$$\sin 0^\circ = 0.$$

Hence, the whole term becomes zero. In a similar manner, all the sine terms become zero and there remains

$$\int_0^{2\pi} y d\beta = 2\pi A_0, \quad \text{or} \quad A_0 = \frac{1}{2\pi} \int_0^{2\pi} y d\beta.$$

But $\int_0^{2\pi} y d\beta$ is the area of the wave for one complete cycle. If the wave is symmetrical with respect to the axis, the two lobes are equal and of opposite sign and $A_0 = 0$. For unsymmetrical waves, A_0 is the distance the axis must be shifted to obtain symmetry. For all purely alternating current waves, $A_0 = 0$.

To obtain the coefficient A_n, Equation (79*a*) should be multi-

plied by cos $n\beta$ and integrated as before. Or,

$$\int_0^{2\pi} y\cos n\beta d\beta = A_0\int_0^{2\pi}\cos n\beta d\beta + A_1\int_0^{2\pi}\cos\beta\cos n\beta d\beta + \cdots$$
$$+ A_n\int_0^{2\pi}\cos^2 n\beta d\beta + B_1\int_0^{2\pi}\sin\beta\cos n\beta d\beta$$
$$+ B_2\int_0^{2\pi}\sin 2\beta\cos n\beta d\beta + \cdots$$
$$+ B_n\int_0^{2\pi}\sin n\beta\cos n\beta d\beta. \qquad (79c)$$

All the terms in Equation (79c) become zero except the term

$$A_n\int_0^{2\pi}\cos^2 n\beta d\beta,$$

whence

$$\int_0^{2\pi} y\cos n\beta d\beta = A_n\int_0^{2\pi}\cos^2 n\beta d\beta.$$

But

$$\cos^2 n\beta = \tfrac{1}{2}(1 + \cos 2n\beta),$$

or

$$A_n\int_0^{2\pi}\cos^2 n\beta d\beta = \frac{A_n}{2}\left[\int_0^{2\pi}d\beta + \int_0^{2\pi}\cos 2n\beta d\beta\right] = \pi A_n,$$

and

$$\int_0^{2\pi} y\cos n\beta d\beta = \pi A_n,$$

or

$$A_n = \frac{1}{\pi}\int_0^{2\pi} y\cos n\beta d\beta.$$

In a similar manner, any of the coefficients B_n may be determined by multiplying by the corresponding sine function sin $n\beta$ and integrating.

From this, the following rule may be written.

First multiply the instantaneous values of (y) by cos $n\beta$, which gives a new curve. The area of this curve, for a cycle, divided by π is the value of A_n.

Since the area divided by 2π is the average ordinate, A_n is twice the average ordinate of the derived curve $y \cos n\beta$ between the limits $\beta = 0$ and $\beta = 2\pi$, or one cycle.

B_n may be obtained in a similar way. From A_n and B_n the angle a_n may be determined.

This method may be applied in the following manner. From the given wave the ordinates are measured every 10 degrees. Each of these ordinates is multiplied by cos n times the value of the cosine of its angle taken from a table and by sin n times the value of the sine of its angle taken from a table. A_n is then twice the average of the cosine values and B_n is twice the average of the sine values. If the two half waves are symmetrical, no even harmonics can exist and these may be omitted from the analysis. The process may be further shortened by taking only a single half wave or 180° of the cycle.

Data. Make an analysis of some alternating wave of electromotive force or current by this method. The analysis should be carried to at least the ninth harmonic.

No. 80. ANALYSIS OF WAVE FORM BY RYAN'S METHOD.

References. Ryan, Vol. 1, Chap. 3; Thompson's "Dynamos," Vol. 2, pp. 72 to 81; Arnold, Vol. 1, pp. 153 to 159; *Elect'n Lond.,* June 28, 1905, John Perry, Wave analysis by graphic reductia; *Elec. Wld. and Eng.,* May 14, 1898, Ranstor and Kennetby, Wave analysis by graphic reductia (see also Experiment 79).

Object. To separate an irregular periodic wave form into its components.

Theory and Method. This method consists in multiplying the periodic curve by sine and cosine functions of the different harmonics chosen. The resultant curves are plotted and from the areas, as measured by a planimeter, the value and phase angle of the component harmonics are determined. The method is based upon the following laws as stated by Professor Ryan:—

"1. The average of the product of any harmonic with any other harmonic is zero, when taken for a complete cycle of the irregular curve."

"2. The average of the product of two sine curves in phase, and having the same frequency, is half the product of their amplitudes."

"3. The average of the product of two sine curves in quadrature, and having the same frequency, is zero."

The method consists in plotting the periodic curve on coordinate paper in rectangular coordinates.

The ordinates should be plotted in percent. of the maximum; that is, all ordinates should be divided by the maximum and plotted. The abscissas should also be plotted to convenient scale. A unit sine curve should be chosen corresponding to the given harmonic to be determined, as the first, third, fifth, etc., with its maximum ordinate unity. Multiply the corresponding ordinates together and plot the curve corresponding to the product. With a planimeter find the difference between the areas of the positive and negative loops of this curve. This area divided by the base, or 360°, gives the average ordinate. This should be multiplied by 2 and called A_n, where (n) is the number of the harmonic chosen.

The same should be repeated with a cosine curve of the same harmonic, and the value of B_n determined. The maximum value of the nth harmonic is then

$$D_n = \sqrt{A_n^2 + B_n^2}.$$

The phase angle may be determined by the relation

$$\tan \beta_n = \frac{B_n}{A_n}.$$

Proceeding in this way, all the component harmonics may be determined.

Data. Analyze some irregular periodic wave by the method outlined above.

No. 81. ANALYSIS OF ALTERNATING CURRENT WAVES BY BLONDEL'S METHOD.

References. Russell, Vol. 2, pp. 112 and 113; Handbuch der Elektrotech., Vol. 2[4], pp. 121 to 122; *Elektrotech. Zeitschr.*, 1900, p. 752, Des Condres, Wave analysis; see also references to Experiment 78.

Object. To derive and analyze an electromotive force or current wave by Blondel's method.

Theory and Method. The indications of a dynamometer type wattmeter are proportional to the product of the currents in the two coils. In order that this may be true, the frequency of the currents in the two coils must be the same and to obtain the maximum indication for given currents, they must be in phase. If we apply the potential wave, the form of which is to be determined, to the terminals of a non-inductive resistance the current and potential waves will have the same form and will be in phase with each other. Pass this current through the fixed coil of a dynamometer type wattmeter of negligible reactance compared with the external resistance. Pass current from a separate machine giving a sine wave, the frequency of which may be varied, through the movable coil. The wattmeter will give large deflections when the separate machine is in synchronism with the fundamental or any of the harmonics in the current wave.

Let the electromotive force wave whose analysis is desired be represented by Fourier's equation

$$f(t) = \Sigma A_n \sin n\omega t + \Sigma B_n \cos n\omega t$$
$$= \Sigma\sqrt{A_n^2 + B_n^2} \sin (n\omega t + \beta_n),$$

where

$$\tan \beta_n = \frac{B_n}{A_n},$$

the wave being alternating. Let the wave of the auxiliary current be represented by

$$i = I \sin (\omega t - \alpha).$$

Then

$$\frac{1}{T}\int_0^T \frac{f(t)}{R} \cdot I \sin (n\omega t - \alpha) dt$$

will be the value of the power in the circuit.

This equals

$$\frac{I}{R} \cdot \frac{1}{T} \int_0^T f(t) \sin (n\omega t - \alpha) dt = \frac{I}{2R} (A_n \cos \alpha - B_n \sin \alpha).$$

If D_1 = the deflection of the wattmeter and
K = its constant,

$$\frac{I}{2R}(A_n \cos \alpha - B_n \sin \alpha) = K^2 D_1.$$

If the phase relation of the current in the movable coil is displaced 90 degrees, the wave will be represented by

$$i = I \cos (n\omega t - \alpha),$$

hence

$$\frac{I}{2R}(A_n \sin \alpha + B_n \cos \alpha) = K^2 D_2,$$

where D_2 is the new wattmeter reading. Hence,

$$\sqrt{A_n^2 + B_n^2} = \frac{2RK^2\sqrt{D_1^2 + D_2^2}}{I}.$$

It is evident that, in order to have these relations hold, the auxiliary machine must be driven from the alternator by some sort of driving device giving speeds varying as the first, third, fifth, etc., harmonic of the electromotive force wave. This is not always easy to accomplish.

Data. Analyze some form of electromotive force wave by the method outlined above.

Curves. Plot the final wave from its harmonics and, if possible, compare it with the same wave as derived by some other method.

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No. 82. ANALYSIS OF WAVE FORM BY PUPIN'S METHOD.

References. Russell, Vol. 2, p. 114; *Am. Jour. Sci.*, May, 1894, Vol. 48, p. 379, M. I. Pupin, Resonance analysis of alternating currents; see also references to Experiment 78.

Object. To determine the amplitude of the various harmonics of an electromotive force wave by a method based upon resonance.

Theory and Method. If a sine wave of electromotive force be applied in a circuit containing resistance, inductance and capacity in series and if the inductance is varied over a wide range, there will be a value of inductance which will produce resonance in the circuit. It has been found that the current in the circuit and the electromotive force across the condenser will be a maximum at this point. If the wave of electromotive force possesses harmonics of an order higher than the first, resonance may be produced for each of these harmonics. Pupin has made use of this to determine the amplitudes of the different harmonics.

An air cored inductance which may be accurately adjusted is connected in series with a condenser. Two electrostatic voltmeters should be used, one being connected across the condenser and the other across the line and the inductance. Keeping the terminal voltage constant, the inductance is varied until the indication of the voltmeter across the capacity is a maximum. This reading will be, approximately, the effective value of the harmonic for which the circuit is in resonance. Other adjustments will give resonance for the fundamental or the other harmonics. It is necessary that the resistance of the inductance coil should be small in comparison with its reactance.

If e_n denotes the nth harmonic of the electromotive force wave,

$$e_n = \frac{1}{C}\int i_n' dt,$$

where i_n' is the current flowing, or

$$e_n = -\frac{D_n}{Cn\omega R}\cos(n\omega t + \beta_n)$$

$$= -\frac{Ln\omega}{R} D_n \cos(n\omega t + \beta_n),$$

where

$$D_n = \sqrt{A_n^2 + B_n^2},$$

and

$$\beta_n = \tan^{-1}\frac{B_n}{A_n}.$$

Then, if E_n = the effective value of e_n,

$$E_n = \frac{Ln\omega D_n}{R\sqrt{2}}$$

and

$$D_n = \frac{\sqrt{2}RE_n}{Ln\omega}.$$

When R is small compared with $L\omega$,

$$n = \frac{1}{2\pi f\sqrt{LC}}, \text{ and}$$

E_n is practically equal to the reading of the electrostatic voltmeter.

Data. Connect up a resonant circuit in which the inductance may be varied. Find the values of L, R and C in the circuit for which the electromotive force across the condenser is a maximum. If the inductance has previously been calibrated, the voltmeter reading across it may be dispensed with.

No. 83. ANALYSIS OF WAVE FORM BY ARMAGNAT'S METHOD.

References. Russell, Vol. 2, pp. 115 to 121; *Ecl. Elec.*, March, 1902, Vol. 30, p. 373, H. Armagnat, Application of the oscillograph to the resonance method; see also references to Experiment 78.

Object. To analyze a wave form by means of an oscillograph.

Theory and Method. An oscillograph may be used in the

analysis of wave form as well as in graphically showing its shape. In this use of the oscillograph the current wave and the electromotive force wave in a resonant circuit are photographed simultaneously on the same plate. The resonant circuit should be one capable of fair adjustment. The height of the current wave will give to scale the value of the amplitude of the harmonic, and the time lag relative to the electromotive force wave may be measured. The principal disadvantage in this method comes in the interference between harmonics producing erroneous values or periodically recurring maxima and minima. The speed of the alternator must be constant or the resultant wave will be difficult to photograph.

Manipulation. The circuit should be arranged with a variable inductance in series with a condenser. This inductance is then varied until resonance is obtained for one of the harmonics. A photograph or tracing is made of the current wave in this circuit. This photograph should show the phase relation of the current wave to the electromotive force wave at the terminals. The constants of the circuit are then determined as well as the scale of the electromotive force and current waves. This should be repeated for other harmonics, including the fundamental.

Data. Analyze an electromotive force or current wave by this method.

Curves. Plot the curve by combining the harmonics and compare it with the curve of the machine as taken by the oscillograph.

No. 84. DERIVATION OF AN ELECTROMOTIVE FORCE WAVE FROM A FLUX WAVE.

Object. To derive the electromotive force wave from the corresponding flux wave.

Theory and Method. In the study of wave forms, as obtained by different methods, it is sometimes desirable to derive an electromotive force wave induced in a coil, corresponding to the wave of flux passing through the coil. The electromotive force in-

duced in a coil is given by the equation

$$e = -k\frac{d\varphi}{dt}, \tag{84a}$$

or, the instantaneous value of the electromotive force is proportional to the slope of the flux curve at that instant.

The curve of flux may be expressed by the equation

$$\varphi = \phi_1 \sin \omega t + \phi_3 \sin 3\omega t + \phi_5 \sin 5\omega t + \phi_7 \sin 7\omega t + \cdots + \phi_n \sin n\omega t. \tag{84b}$$

Differentiating this equation and substituting the values thus obtained in Equation (84a),

$$e = -k\frac{d\varphi}{dt} = -k\omega[\phi_1 \cos \omega t + 3\phi_3 \cos 3\omega t + 5\phi_5 \cos 5\omega t + 7\phi_7 \cos 7\omega t + \cdots + n\phi_n \cos n\omega t], \tag{84c}$$

or, the electromotive force wave is much more distorted than the flux wave. It is also seen that the electromotive force components are at 90° to the flux components; that is, a flat topped wave of flux will correspond to a peaked wave of electromotive force.

If the wave of flux is given, the electromotive force wave may be derived by dividing the flux wave into small areas by means of ordinates equally spaced. The electromotive force at the instant denoted by the middle ordinate of each of these areas is proportional to the average slope of the curve during that interval. Each portion of the flux wave parallel to the axis of time corresponds to a zero point on the electromotive force wave. Every change from positive to negative slope represents a point where the electromotive force curve changes from a positive to a negative value, or vice versa.

Curves. From a flux wave, obtained by construction or by some method such as the one explained in Experiment 74, derive the corresponding electromotive force wave.

Question. Which is the leading wave, and why?

No. 85. DERIVATION OF A FLUX WAVE FROM AN ELECTROMOTIVE FORCE WAVE.

Object. To derive a flux wave from an electromotive force wave, by construction.

Theory and Method. In studying iron losses, it is often desirable to know the flux wave. Most methods of deriving waves give the electromotive force wave. The following method of construction may be used to derive the corresponding flux wave.

Since the electromotive force induced in a coil at any instant is given by the equation

$$e = -n\frac{d\varphi}{dt}, \tag{85a}$$

$$edt = -nd\varphi,$$

or

$$\int_{t_1}^{t_2} edt = -N\int_{\phi_1}^{\phi_2} d\varphi.$$

Assume that

t_1 = the time when the flux is zero,

and

t_2 = the time when the flux is a maximum.

Then

$$\int_{t_1}^{t_2} edt = -N\phi. \tag{85b}$$

From Equation (85*b*) it is seen that the maximum flux is proportional to the sum of the areas obtained by multiplying the instantaneous values of the electromotive force and each time interval between the time when the flux is zero and when it is a maximum. Or, the flux ϕ is proportional to the average electromotive force multiplied by $(t_2 - t_1)$.

The flux is a maximum when the electromotive force is zero. Hence, t_2 is readily located on the curve of electromotive force. The position of t_1 is determined from the following. The positive and negative values of ϕ are equal. Since each is proportional to part of the electromotive force wave, the two parts are equal.

Hence, t_1 is at the point dividing the electromotive force area into equal parts.

The wave is then derived as follows. The electromotive force wave is divided by ordinates spaced 5° or 10° apart. The maximum and zero positions of the flux wave are then located as previously explained. Starting at the zero point, the value of each area is computed and their sum taken from the zero to the maximum points. The maximum ordinate is proportional to this sum and the other ordinates are proportional to the area to the point corresponding to that ordinate.

Curves. From some electromotive force wave, obtained by means of the oscillograph or by some contact method, derive the flux wave.

Question. Which is the leading ordinate, and why?

No. 86. GRAPHICAL STUDY OF THE EFFECT OF IRON ON THE WAVE SHAPE OF CURRENT AND ELECTROMOTIVE FORCE.

Object. To construct the wave of current from the hysteresis loop and a given wave shape of flux.

Theory and Method. To gain an understanding of the peculiarities in ratios in three phase transformation and to learn how iron in the magnetic circuit causes wave distortion in general, it is necessary to know the relative shapes of the waves of current and electromotive force in a theoretical circuit. For this reason a method is given here for deriving the different curves. In order to derive these curves, a hysteresis loop of the steel in the core, as of a transformer, should be determined by one of the methods described in Vol. I, Direct Currents.

The hysteresis loop for this experiment is most conveniently plotted between flux ϕ and exciting current I for the density corresponding to the electromotive force to be studied. Reduce the values of flux ϕ and current I to percentages of the maximum

flux and current. Assume first a sine wave of electromotive force which gives a wave form of flux similar but at 90 degrees from

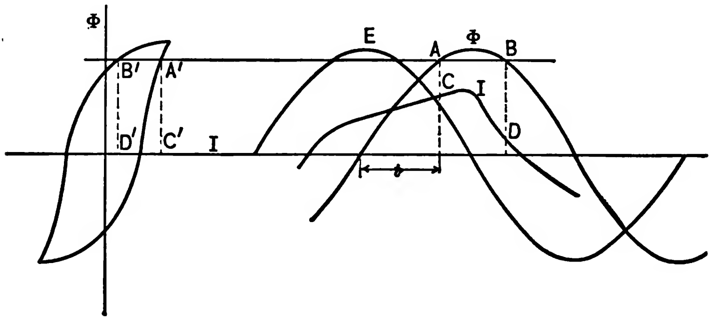

FIG. 86*A*. Construction of the current curve of a transformer corresponding to sine waves of flux and electromotive force and a given hysteresis loop.

it. Reduce these values to percentage of maximum; that is, plot the sine curve, Figure 86*A*, using values taken from a table of sines. Plot the ordinates to the same scale as those of the hysteresis loop. At some instant of time (t) when the flux is increasing, it will reach a value A on the flux wave. The current at that instant is given by projecting the point A on to the hysteresis loop at A' The current will be C'. This value is the current at the instant (t) and should be laid off on the ordinate A. Similarly, the point B on the descending portion of the curve is chosen and the corresponding point D on the current curve is determined. Proceeding in this manner, the current curve is plotted. It will

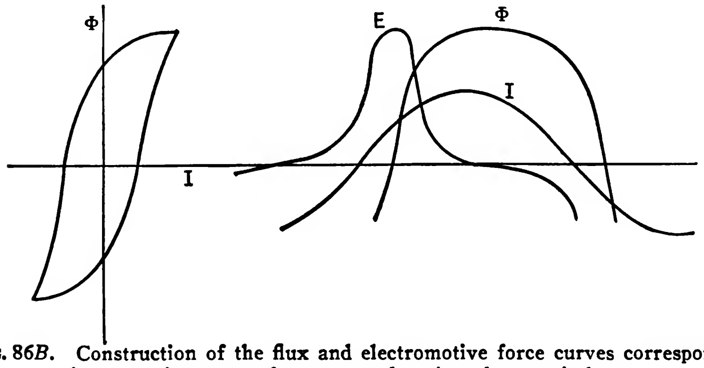

FIG. 86*B*. Construction of the flux and electromotive force curves corresponding to a sine wave of current and a given hysteresis loop.

be seen that the current curve corresponding to the sine wave of electromotive force and flux, is distorted. By analysis, it will be found to have a large first and a large third harmonic, with the fifth harmonic also present. Different values of the saturation give different amounts of distortion.

In a similar manner, a sine curve of current passed through a coil containing iron gives distorted waves of flux and electromotive force. It will be found that the flux wave is flat topped and that the electromotive force wave is quite peaked. The distortion caused by a triple is three times as pronounced in the electromotive wave as it is in the corresponding flux wave. The construction for such a case is shown in Figure 86*B*.

Data. Obtain a hysteresis loop of the steel at the required density.

Curves. Derive the current wave corersponding to a sine wave of electromotive force. Derive a flux wave corresponding to a sine wave of current. From this flux wave, derive the electromotive force wave by the method described in Experiment 84.

No. 87. STUDY OF A QUARTER-PHASE SYSTEM.

References. Steinmetz' "A.C. Phenomena," Chap. 33, 35, 37 and 39; Russell, Vol. 1, Chap. 12; Karapetoff, Chap. 20; Franklin and Esty, Chap. 6; Thompson's "Polyphase," Chap. 3; Thomälen, Chap. 16; Arnold, Vol. 1, p. 266; Arnold, Vol. 2, p. 305.

Object. To study the voltage and current relations in a quarter-phase system.

Theory and Method. A quarter-phase system is one in which the electromotive forces are at the same frequency and at 90° from each other in two circuits. It may be produced by a quarter-phase generator in which the two phases are either independent or are connected at some common point.

If the phases are entirely independent, there is no voltage between them. In such a system a peculiar connection of trans-

formers or apparent ground may give peculiar voltage conditions between phases. Such a system may be loaded as two single phase systems or it may be used to produce other forms of polyphase systems by the use of transformers. It is useful chiefly in the operation of alternating current motors.

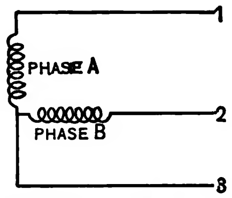

FIG. 87*A*. Three wire quarter phase system.

The phases may be connected to produce a quarter-phase three-wire system as shown in Figure 87*A*. If the voltage from 1 to 3 and from 2 to 3 is E, the voltage from 1 to 2 is $\sqrt{2}\,E$. The current in 3 is $\sqrt{2}$ times the current in either 1 or 2, with balanced loading.

The two phases may be connected as in Figure 87*B*, known as a "star" connection or they may be connected as shown in Figure 87*C*, known as a "mesh" connection; or in any one of several combinations which may suggest themselves.

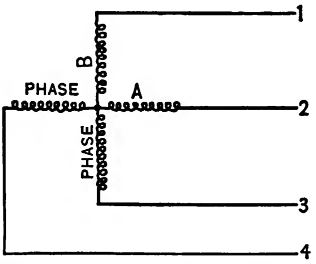

FIG. 87*B*. Star connection of a quarter phase system.

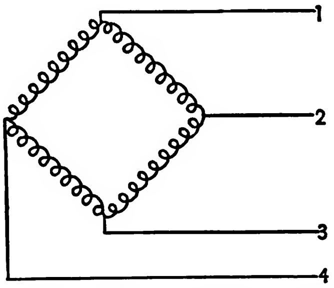

FIG. 87*C*. Mesh connection of a quarter phase system.

Data. Study the voltage and current relations in each of the above quarter-phase systems, using non-inductive and inductive loading.

Curves. Draw vector and phase diagrams of the quantities measured.

No. 88. MEASUREMENT OF POWER IN A QUARTER-PHASE SYSTEM.

References. Franklin and Esty, pp. 110 to 116; Karapetoff, Chap. 20; Steinmetz' "A.C. Phenomena," Chap. 34; Russell, Vol. 1, Chap. 12; Esty, pp. 106 to 111; Thomälen, Chap. 16; Thompson's "Polyphase," Chap. 15.

Object. To measure power in a quarter-phase system.

Theory and Method. A quarter-phase system may be considered as two single phase systems from the standpoint of the measurement of power. Care must always be taken to have the potential coil of each wattmeter connected across the same phase in which the current coil is connected and not between phases or across the wrong phase. If the pressure coil be connected in one phase and the current coil in the other, the wattmeter will read zero on non-inductive load. The sine of the angle of lag between current and electromotive force in the circuit in which the current coil is connected may be determined by dividing the wattmeter reading in this case by the volt-amperes. When properly connected, the total power is the sum of the two wattmeter readings.

Data. Measure the power absorbed in a quarter-phase system with non-inductive load and also when loaded with an induction motor. Determine the power factor.

Question. How could you get a direct reading of the reactance power in a balanced quarter-phase system, using one wattmeter?

No. 89. THREE WATTMETER METHOD OF MEASURING POWER IN A THREE PHASE SYSTEM.

References. Karapetoff, Chap. 20; Steinmetz' "A.C. Phenomena," Chap. 34; Franklin and Esty, pp. 110 to 116; Russell, Vol. 1, Chap. 11; Esty, pp. 106 to 113; Thomälen, Chap. 16; Arnold, Vol. 1, Chap. 16; Handbuch der Elektrotech., Vol. 2, pp. 138 to 144; Thompson's "Polyphase," Chap. 15.

Object. To study the three wattmeter method of measurement of power in a three phase system.

Theory and Method. In a three phase system with neutral return, it is necessary to use three wattmeters in order to be able to measure the power accurately on unbalanced loads. The connections for three wattmeters are shown in Figure 89*A* and the phase relations in the two coils are represented in Figure 89*B*. It will be seen that this is equivalent to three wattmeters in three single phase circuits with one wire used as a common return.

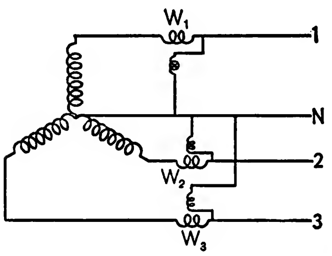

FIG. 89*A*. Connections for the three wattmeter method of measuring power in a three phase system.

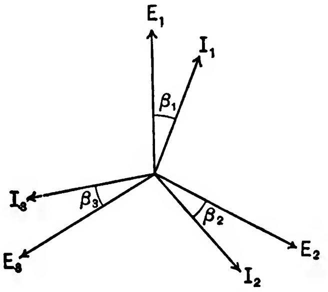

FIG. 89*B*. Vector relations of current and electromotive force in a three phase circuit.

In a balanced three phase system, however, the neutral carries no current.

The total power used is the sum of the three wattmeter readings. If the load is balanced, the three wattmeters will all read alike. In that case only one wattmeter is necessary and it may be connected as W_1, W_2 or W_3 in Figure 89*A*. The true power

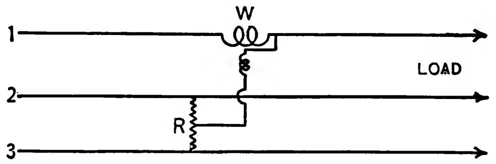

FIG. 89*C*. Measurement of power in a three phase system by means of one wattmeter.

is then three times the reading of this meter or its scale may be graduated to read total power directly. A more accurate method,

requiring the use of a single wattmeter, is that shown in Figure 89*C*, where a *Y* box is employed. *R* is a resistance twice the value of the resistance of the potential coil circuit of the wattmeter. The potential coil circuit is connected to the middle of this resistance for a neutral point. The power in the circuit is three times the wattmeter reading.

Data. Measure power in a three phase four-wire circuit by means of three wattmeters. Determine the power factor of the load.

No. 90. TWO WATTMETER METHOD OF MEASURING POWER IN A THREE PHASE SYSTEM.

References. Karapetoff, Chap. 20; Steinmetz' "A.C. Phenomena," Chap. 34; Franklin and Esty, pp. 110 to 116; Russell, Vol. 1, Chap. 11; Esty, pp. 106 to 113; Thomälen, Chap. 16; Arnold, Vol. 1, Chap. 16; Handbuch der Elektrotech., Vol. 2, pp. 138 to 144; Thompson's "Polyphase," Chap. 15; *Elec. Wld. and Eng.*, February 3, 1900, Budd Frankenfield, Power factor measurements.

Object. To measure power in a three phase circuit by means of two wattmeters.

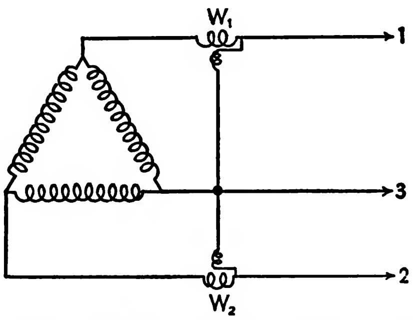

FIG. 90*A*. Connections for the two wattmeter method of measuring power in a three phase circuit.

Theory and Method. From the deflections of two standard wattmeters, the power in a three phase circuit may be deter-

mined. The connections are made as shown in Figure 90*A*, the current coils of the wattmeters being connected in two of the lines 1 and 2 while the pressure coils are connected across the lines 1 and 3 and the lines 2 and 3, respectively. Figure 90*B* shows the phase relations between the currents in the current and pressure coils in each wattmeter for various types of loads. It will be seen that, for non-inductive loads, the current in the current coil lags 30 degrees behind the current in the pressure coil in W_1 and that the current in the current coil leads the current in the pressure coil by 30 degrees in W_2. For an inductive load, shown by a lag angle of β', where $\cos\beta'$ is the power factor of the load, the current in the current coil of W_1 is $30° + \beta'$ behind

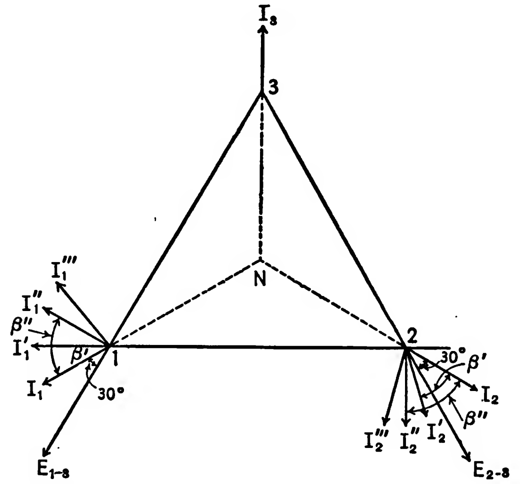

FIG. 90*B*. Vector diagram for the two wattmeter method.

the current in the pressure coil, while it is $30° - \beta'$ ahead of the current in the pressure coil in W_2. The deflections will then be

$$P_1 = E_{1-3}I_1 \cos(30° + \beta'), \qquad (90a)$$

$$P_2 = E_{2-3}I_2 \cos(30° - \beta'), \qquad (90b)$$

or

$$P_1 = E_{1-3}I_1(\cos 30° \cos\beta' - \sin 30° \sin\beta'),$$

$$P_2 = E_{2-3}I_2(\cos 30° \cos\beta' + \sin 30° \sin\beta').$$

If the load is balanced,

$$E_{1-3} = E_{2-3} = E,$$
$$I_1 = I_2 = I,$$

or

$$P_1 + P_2 = 2EI \cos 30^\circ \cos \beta'$$
$$= \sqrt{3} EI \cos \beta',$$

the true power in a balanced three phase circuit. The same may be proved for an unbalanced load. Hence, the sum of the two wattmeter deflections is the true power in a three phase circuit.

From an inspection of the Figure 90*B*, it will be seen that, when $\beta' = 0$, which is the condition for non-inductive load, the two deflections are the same. When $\beta' = 60^\circ$, one wattmeter reads zero, and when β' is greater than 60°, one wattmeter reads negative and its deflections should be subtracted from those of the other wattmeter for the true power in the circuit. The relative value of the two deflections is a measure of the power factor in the circuit. The power factor may be determined as follows:—

$$P_2 + P_1 = 2EI \cos 30^\circ \cos \beta',$$
$$P_2 - P_1 = 2EI \sin 30^\circ \sin \beta',$$

hence,

$$\frac{P_2 - P_1}{P_2 + P_1} = \tan 30^\circ \tan \beta'$$

or

$$\tan \beta' = \sqrt{3} \frac{P_2 - P_1}{P_2 + P_1},$$

or the lag angle, and hence the power factor, may be determined by the relation of the wattmeter readings.

In using two wattmeters for the measurement of power in three phase circuits having low power factor, it is important to have the wattmeters connected in the circuit with the current and pressure coils in the proper relation so that there will be no doubt as to whether the smaller reading is to be added or subtracted. This may be done by connecting both meters with their coils in

the same relative positions. In the test of an induction motor operating from no load to full load, it will probably be necessary to reverse one of the instruments during the test.

Data. Measure the power in some three phase circuit by means of two wattmeters. An induction motor operated through its full range of load forms an excellent circuit for this purpose. Determine the power factor of the circuit by the relation of the total volt-amperes to the watts and also from the relation of the two wattmeter readings.

Question. How could you get a direct reading of the reactive power in a balanced three-phase system, using one wattmeter?

No. 91. ONE WATTMETER METHOD OF MEASURING POWER IN A BALANCED THREE PHASE SYSTEM.

References. Karapetoff, Chap. 20; Steinmetz' "A.C. Phenomena," Chap. 34; Franklin and Esty, pp. 110 to 116; Russell, Vol. 1, Chap. 11; Esty, pp. 106 to 113; Thomälen, Chap. 16; Arnold, Vol. 1, Chap. 16; Handbuch der Elektrotech., Vol. 2, pp. 138 to 144; Thompson's "Polyphase," Chap. 15.

Object. To measure the power in a balanced three phase circuit by means of one wattmeter.

Theory and Method. One method of measuring power in a three phase circuit by means of one wattmeter was mentioned in Experiment 89. This is used principally in watt-hour meters. Another single wattmeter method, based upon the two wattmeter method shown in Experiment 90, is described below.

Figure 91*A* shows the connections for this method of measure-

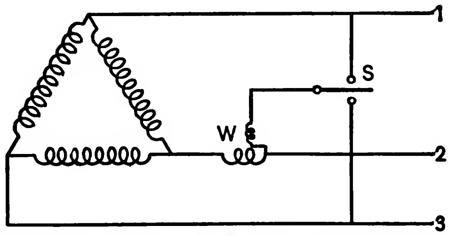

FIG. 91*A*. One wattmeter method of measuring power in a three phase circuit.

ment. The current coil of the wattmeter is connected in one line, as 2. The pressure coil may be connected either from 2 to 1 or from 2 to 3 by means of the double throw switch S. Readings are taken with the switch thrown first in one direction and then in the other, for each load upon the circuit.

From Figure 91*B*, it will be seen that

$$P_1 = EI\cos(30° + \beta),$$

and

$$P_3 = EI\cos(30° - \beta),$$

or

$$P_1 + P_3 = \sqrt{3}EI\cos\beta,$$

as shown for the two wattmeter method in Experiment 90. This single wattmeter method is but a modification of the two watt-

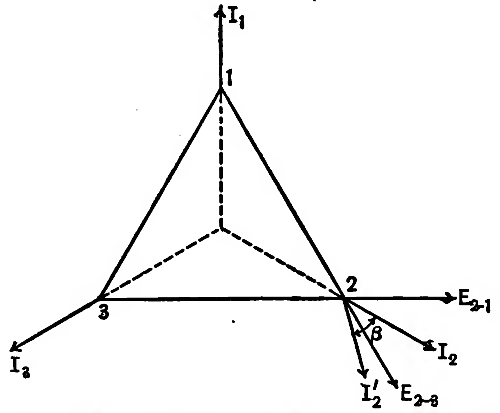

FIG. 91*B*. **Vector diagram for the one wattmeter method.**

meter method and the equations in Experiment 90 apply to this case also. It is evident that this method applies only to a balanced load.

Data. Measure the power in some balanced three phase circuit by means of one wattmeter. An induction motor operated through its full range of load forms an excellent circuit for this purpose. Determine the power factor of the circuit by the relation of the total volt-amperes to the watts and also from the relation of the two wattmeter readings.

No. 92. CONNECTION AND STUDY OF TRANSFORMERS IN A QUARTER PHASE SYSTEM.

References. Thompson's "Polyphase," pp. 54 to 60; Karapetoff, pp. 452 to 462; Esty, pp. 222 to 224; Steinmetz' "A.C. Phenomena," p. 592; Russell, Vol. 2, Chap. 10; *Min. and Sci. Pr.*, February 7, 1904, W. H. Kritzer, Ordinary transformer connections; *Prac. Eng.*, December 11, 1908, Serial, Transformer connections; *Elec. Rev. Lond.*, December 11, 1908, The parallel connections of three phase transformers; *Elec. Wld.*, December 5, 1908; W. T. Ryan, Star and delta transformer connections.

Object. To study the connection of transformers in a quarter phase system.

Theory and Method. In this country the three phase system is used for transmission purposes in preference to the quarter phase system. Since there are no even harmonics in the electromotive force wave of an alternator, the inter-connection of the transformer primaries at the neutral of a quarter phase system can produce no appreciable effect upon the shape of the electromotive force wave across the coils, as it may in the case of a third harmonic in a three phase star system. The quarter phase system is usually operated with two independent circuits. It may then be classed as two independent single phase circuits from the same machine and transformer connections would not alter the wave form of electromotive force unless there is a considerable resistance in the leads to the transformer.

Data. Connect transformers in a quarter phase system in the following manner and measure the electromotive force in all possible combinations.

1. Independent systems, with phases entirely separate.
2. Three-wire quarter phase.
3. Four-wire quarter phase, with the neutrals of primaries connected.
4. Four-wire quarter phase, with the neutrals of secondaries connected.

5. Four-wire quarter phase, with the neutrals of both primaries and secondaries connected.

In reporting, show reasons for the results found, illustrating with diagrams.

No. 93. STUDY OF A THREE PHASE SYSTEM.

References. Thompson's "Polyphase," pp. 54 to 60; Karapetoff, pp. 452 to 462; Esty, pp. 222 to 224; Steinmetz' "A.C. Phenomena," Chap. 36; Russell, Vol. 2, Chap. 10; *Min. and Sci. Pr.,* February 7, 1904, W. H. Kritzer, Ordinary transformer connections; *Prac. Eng.,* December 11, 1908, Serial, Transformer connections; *Elec. Wld.,* February 9, 1907, A. S. McAllister, Three phase transformation; *Trans. Am. Inst. Elec. Eng.,* April, 1907, John S. Peck, Relative advantage of single and three phase transformers; H. W. Foliey, Relative merits of single and three phase transmission.

Object. To study a three phase system under load.

Theory and Method. The general laws of non-inductive loads in three phase systems, are considered here. The problem of inductive loads will be studied in connection with the transformer experiments which follow. Unbalanced loads at the end of a line may produce unbalanced electromotive force relations. The three voltages will form the sides of a triangle, the intersecting medians of which give the neutral.

In a loaded "star" connected system, the voltages across the three branches adjust themselves so that the power expended in the system is a minimum. In a loaded "delta" connected system, the instantaneous value of the current in any line is equal to the sum of the instantaneous values of the currents in the two branches connected to that line. From this fact and knowing the values of the resistances in the branches, simultaneous equations may be written from which the various electromotive forces may be derived.

Data. Make several combinations of equal and unequal resistances in a three phase circuit. Study the voltage and current relations by means of meters. Draw diagrams illustrating the circuits.

No. 94. STAR CONNECTION OF TRANSFORMERS.

References. Berg, pp. 139 and 140; Karapetoff, pp. 466 to 470; Esty, pp. 223 to 224; Steinmetz' "A.C. Phenomena," Chap. 36; Russell, Vol. 2, Chap. 10; *Trans. Am. Inst. Elec. Eng.*, July, 1903, F. O. Blackwell, Star and delta connections of alternators; *Min. and Sci. Pr.*, February 7, 1904, W. H. Kritzer, Ordinary transformer connections; *Prac. Eng.*, December 11, 1908, Serial, Transformer connections.

Object. To determine the voltage and current relations of transformers in "star" and in "open delta" connection.

Theory and Method. In balanced three phase systems, neither triple harmonics nor their multiples can exist in the electromotive forces between wires; neither can they exist in the currents in the lines. These triple harmonics may exist between lines and neutral. It is the purpose of this experiment to study the effects of these laws on the operation of a three phase transformer circuit. In Experiment 86 it was seen that, with iron in an alternating current magnetic circuit, either triple frequency voltage or triple frequency currents, must exist.

Consider three transformers with their primaries star connected, the electromotive force between lines being a sine wave. If a sine wave of current flows in the coils, a peaked wave of electromotive force must exist between the neutral and each outside wire. The effective value of this electromotive force wave is greater than that of a sine wave. Hence, since the triple harmonics in the peaked waves oppose each other between lines, the voltage to neutral will be greater than the line voltage divided by the square root of three. If the secondaries are star connected, the same ratio per coil and per line will exist as would be the case

if these triple harmonics were not present. A peaked electromotive force wave corresponds with a flat topped flux wave, as was seen in Experiment 84. This, in turn, affects the losses in the transformer.

With the secondary connected in open delta, a voltage greater than $\frac{E}{\sqrt{3}}$ will exist across each coil, where E is the line voltage due to the existence of the triple harmonic. Across the opening of the delta, a value of electromotive force will exist which is equal to one fundamental and twice the triple harmonic. If one corner of an ordinary delta is opened, a voltage equal to three times the triple harmonic, will be found.

These relations give a means of measuring the value of the triple harmonic in a three phase circuit. Connect the primaries in star and their secondaries in delta, with one corner open. Measure the value of the voltage across this opening. The triple harmonic in the secondary is one third of this voltage. If the voltage across one side of the secondary is taken, the fundamental may be determined, as

$$E_1 = \sqrt{E^2 - E_3^2},$$

where E_1 = the effective value of the fundamental,

E_3 = the effective value of the triple,

E = the effective value of the voltage on one side of the delta.

The voltage across an open delta should be

$$E' = \sqrt{E_1^2 + (2E_3)^2}.$$

The value of the triple harmonic in the primary may be found by multiplying the triple harmonic in the secondary by the ratio of transformation. It may also be found by a comparison of the line voltage and the voltage to neutral.

Data. Connect three transformers in a three phase circuit with their primaries in "star" and with their secondary circuit open at one corner of a delta connection. Determine the value

of the triple harmonic and the values of the voltage ratios in this arrangement. Take similar readings with both the primaries and the secondaries star connected. Take data with the primaries star connected and with the neutral connected to the neutral of the system furnishing the voltage. Explain the reasons for any differences in ratios of neutral to line voltage. Determine the exciting current for one transformer at normal voltage before connecting into the star. Measure the current in each line when star connected. Explain results.

No. 95. DELTA CONNECTION OF TRANSFORMERS.

References. Berg, pp. 139 and 140; Karapetoff, pp. 466 to 470; Esty, pp. 223 to 224; Steinmetz' "A.C. Phenomena," Chap. 36; Russell, Vol. 2, Chap. 10; *Trans. Am. Inst. Elec. Eng.*, July, 1903, F. O. Blackwell, Star and delta connections of alternators; *Min. and Sci. Pr.*, February 7, 1904, W. H. Kritzer, Ordinary transformer connections; *Prac. Eng.*, December 11, 1908, Serial, Transformer connections.

Object. To study voltage ratios in three phase delta connected transformers.

Theory and Method. After the secondary delta with one open corner has been formed, the primaries being in star, this corner should be closed through an ammeter and the value of current produced by the triple voltages in this path measured. It will be found that this current is relatively low, that the voltage across each side of the delta changes and that the voltage from line to neutral has become $E/\sqrt{3}$, where E is the line voltage. The cause for this must be the small current flowing in the delta. If measured, this current will be found to be the value of the triple frequency component required by the presence of the iron when a sine wave of electromotive force is impressed, as shown in Experiment 86. It makes no difference which coil carries this triple frequency component; it will be inversely in the ratio of

transformation, according to which coil is used, provided the iron is at the same saturation.

To further prove this fact, the primaries should be connected in delta and the secondaries in delta with one corner open. The voltage across this corner should be measured. If the circuit is balanced, this voltage will be found to be practically zero. Closing this corner may cause a slight current to flow, which current produces part of the magnetizing effect.

Data. Connect the transformers with primaries in star and secondaries in delta with one corner open. Measure the voltages in the transformers and that across the open corner and measure the currents in the primaries. Close the corner through an ammeter, and repeat the readings, taking the value of the triple frequency circulating current. Connect again with the primaries in delta and repeat the readings as described above. Explain results.

No. 96. TRANSFORMATION OF THREE PHASE TO QUARTER PHASE AND QUARTER PHASE TO THREE PHASE BY SCOTT'S CONNECTION.

References. Berg, pp. 141 and 142; Karapetoff, pp. 476 to 478; Thompson's "Polyphase," p. 334; Esty, pp. 225 to 230; Steinmetz' "A.C. Phenomena," p. 668; Arnold, Vol. 2, pp. 115 to 120; Russell, Vol. 1, pp. 261 to 265; Russell, Vol. 2, pp. 279 and 280; *Min. and Sci. Pr.,* February 7, 1904, W. H. Kritzer, Ordinary transformer connections; *Prac. Eng.,* December 11, 1908, Serial, Transformer connections.

Object. To show how phase transformation may be accomplished in an alternating current system.

Theory and Method. Phase transformation in an alternating current system is sometimes necessary. If a revolving field can be produced in an iron structure common to two windings, this may be easily accomplished. Usually it is necessary to make these changes by means of single phase transformers.

In the equilateral triangle ABC, Figure 96A, the perpendicular BD bisects the side AC at D and the following relation exists:—

$$BD = AC\frac{\sqrt{3}}{2},$$
$$= 0.866AC.$$

If the secondaries AC and BD of two transformers are connected as shown in Figure 96B, and if their primaries are supplied from a quarter phase system, the points A, B and C will form the terminals of a three phase system, provided the pressure across BD is 0.866AC and provided D is the middle point of AC. The two transformers are usually built alike, with extra taps from the middle and the 0.866 points.

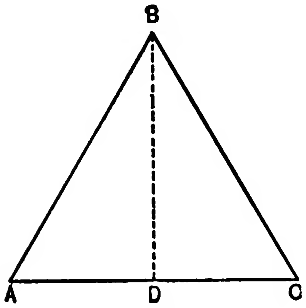

FIG. 96A. Equilateral triangle showing derivation of Scott's method.

The same system may be used to transform from three phase to quarter phase by using A, B and C as primary terminals.

In a balanced system the transformers will be equally loaded, since, for non-inductive load (Figure 96C)

$$P_{BD} = E_{BD} I_B,$$

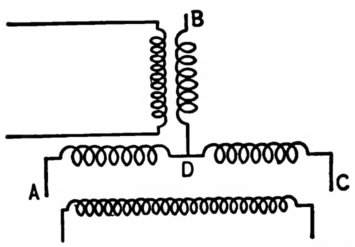

FIG. 96B. Connections for transforming from two phase to three phase by Scott's method.

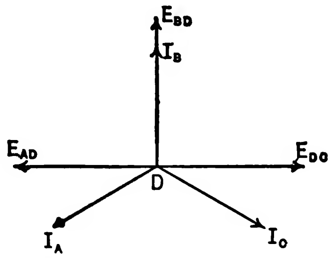

FIG. 96C. Vector's diagram for Scott's method.

and

$$P_{AC} = 2E_{DC}I_C \cos 30°.$$

But

$$E_{BD} = 0.866E_{AC},$$

and

$$\cos 30° = 0.866.$$

Hence

$$P_{BD} = P_{AC},$$

and

$$I_{BD} = I_{AC},$$

or, the heating and rating will be the same. Inductive loads will cause a slight unbalancing of the system.

Data. Connect two transformers by the Scott method and test under different types of load.

No. 97. TRANSFORMATION FROM THREE PHASE TO SIX PHASE.

References. Karapetoff, pp. 478 to 480; Thompson's "Dynamos," Vol. 2, p. 336; Steinmetz' "A.C. Phenomena," pp. 636, 639, 669 and 670; Arnold, Vol. 2, pp. 308 to 311; Franklin and Esty, pp, 230 to 232; Esty, pp. 302 to 305; McAllister, pp. 175 to 180; *Am. Elect'n,* December, 1902, A. S. McAllister, Six phase transformation; *Elec. Wld.,* May 19, 1906, G. J. Reynolds, Three to six phase transformation and connections to rotary converters.

Object. To change from three phase to six phase, by means of transformers.

Theory and Method. In the operation of synchronous converters, it is found advisable to employ a six phase circuit, as the rating of the machine is increased by so doing. Since most transmission systems are three phase, it becomes necessary to transform from three phase to six phase.

When each of the transformers has two secondary windings, they may be connected together in double delta as shown in

Figure 97*A*. The points 1, 2, 3, 4, 5 and 6 are, in this case, directly connected to the slip rings of the six phase synchronous

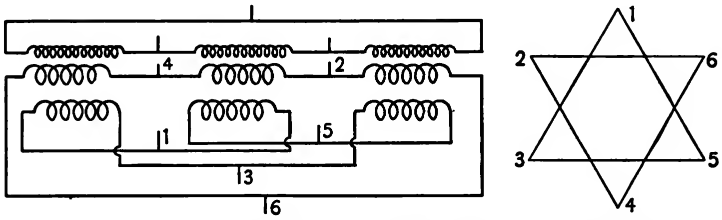

FIG. 97*A*. Transformation from three phase to six phase delta.

converter. Another method is the one shown in Figure 97*B*, where but three secondary coils are used. In this case the delta

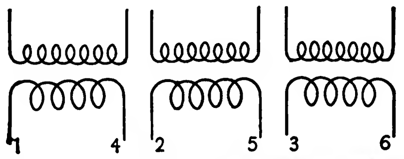

FIG. 97*B*. Transformation from three phase to six phase diametral connection.

is formed by the winding of the armature of the synchronous converter. Where the neutral is required for a three wire system

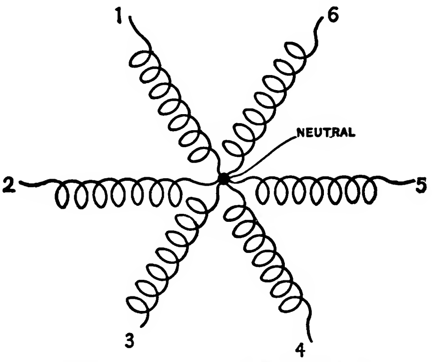

FIG. 97*C*. Transformation from three phase to six phase star.

on the direct current side, the neutrals of the secondaries may be connected together and the neutral wire connected to this point, as in Figure 97*C*. This requires either an extra tap at the center

of each secondary, or two secondaries connected in series outside the cases.

Data. Connect three transformers to transform from three phase to six phase and take the necessary readings. Discuss the relative merits of each method.

No. 98. CONNECTION OF THE ARMATURE COILS OF A POLYPHASE ALTERNATOR.

References. Karapetoff, Chap. 26; Thompson's "Dynamos," Vol. 2, Chap. 6; Thompson's "Polyphase," Chap. 4; Arnold, Vol. 3; Handbuch der Elektrotech., Vol. 4, pp. 52 to 73; Parshall and Hobart, Armature windings; Esty, pp. 96 to 106 and 113 to 123; *Elec. Jour.*, June, 1905, F. D. Newbury, Armature windings of alternators.

Object. To study the methods of connection of armature coils of an alternator.

Theory and Method. In performing this experiment, it is desirable to have a revolving field type of alternator with at least six equal coils symmetrically arranged about the armature. The terminals of these coils should be brought out independent of each other, so that various combinations may be made.

It is desired to study the resultant vectors produced by combinations of equal vectors at different phase angles with each other.

Data. Find the voltage across each coil, the field excitation and the speed of the machine being normal. Make connections successively for three phase star, three phase delta, single phase and quarter phase. Make other combinations and, in the report, give the vector diagrams showing the combinations tried.

Question. Is the rating of the machine affected by the method of connection of the armature coils, and if so, to what extent?

No. 99. REGULATION OF A POLYPHASE ALTERNATOR UNDER LOAD.

References. Thompson's "Polyphase," pp. 391 and 392; Steinmetz' "A.C. Phenomena," Chap. 22; Arnold, Vol. 4, pp. 39, 40 and 56 to 58.

Object. To determine the regulation of a polyphase alternator.

Theory and Method. In this experiment one of the methods described in Experiments 54, 55, 56, 57, 58 and 59 should be used. The short circuit test should be made with the armature connected polyphase and not single phase, care being taken that the leads have equal resistances in order to insure balanced conditions.

It must be remembered that, if nI is the value of the ampere-turns per phase, $\sqrt{2}nI$ is the armature reaction of a quarter phase machine and $1.5\sqrt{2}nI$ is the armature reaction for a three phase machine.

Data. Secure data for the open circuit and load saturation curves of the machine connected polyphase. Compute the regulation by one of the methods previously described.

No. 100. SYNCHRONOUS IMPEDANCE OF A POLYPHASE ALTERNATOR ARMATURE.

References. Arnold, Vol. 4, pp. 39, 40 and 56 to 58; see also references to Experiment 50.

Object. To determine the synchronous impedance of a polyphase alternator armature.

Theory and Method. The measurements for the synchronous impedance of a polyphase alternator should be taken as described in Experiment 50. Care should be taken to have the impedance in all the short circuiting leads equal, in order to insure equal currents in all the armature coils.

Since the value of the armature reaction in a single phase alternator varies from zero to $\sqrt{2}nI$ twice during each cycle and since it is constant for a polyphase machine, being $\sqrt{2}nI$ for a quarter

phase machine and $1.5\sqrt{2}nI$ for a three phase machine, the value of the field current to produce the same armature current in single phase and polyphase machines will not be the same. For instance, if a quarter phase machine has only one of its phases short circuited, it will require a field current I_f for a given armature current. If both phases are short circuited, it will require a field current I_f' to produce the same armature current in each coil, where $I_f' = \sqrt{2}I_f$.

Data. Determine the value of the synchronous impedance of an alternator armature when connected single phase and again when connected polyphase, and compare these results.

No. 101. PARALLEL OPERATION OF POLYPHASE ALTERNATORS.

References. Arnold, Vol. 4, pp. 480 to 485 and Chap. 19; see also references to Experiment 65.

Object. To operate two or more polyphase alternators in parallel.

Theory and Method. The method of synchronizing is that described in Experiments 65 and 66. In the case of a polyphase machine which has not previously been operated in parallel with a given polyphase system, the phase rotation must be determined. This may be accomplished by the use of an induction motor or by the use of lamps. The induction motor should first be connected to the bus-bars when supplied with power from one of the machines which has previously been in service, and its direction of rotation determined. The first machine should then be disconnected from the bus-bars and the second machine should be connected to them. If the induction motor runs in the same direction, the machines have the same phase rotation and may be operated in parallel. If the second machine has the wrong phase rotation, one of its phases should be reversed. Another method is by the use of lamps, three of which should be employed in the

case of a three phase system. If all three lamps beat together, the phase rotation is correct. If the neutrals in a three phase system are to be connected, the wave form should first be determined to see if the triple harmonics are in phase. After making connections, it is well to determine the value of the neutral current both when each machine is normally excited and when the excitation of one or both machines is not normal, as the neutral current may be sufficient to burn out the armature of one of the machines.

Data. Determine the phase rotation of two polyphase alternators, and connect them in parallel. Repeat the tests described in Experiments 65 and 66. If the neutrals are connected, determine the triple frequency neutral current when the machines are normally excited and also when the excitation of one or both of the machines is not normal.

No. 102. OPERATION OF POLYPHASE SYNCHRONOUS MOTORS.

References. Arnold, Vol. 4, pp. 488 to 491; see also references to Experiment 65.

Object. To operate a polyphase synchronous motor.

Theory and Method. The polyphase synchronous motor is self starting if supplied with polyphase alternating current power, but has very little starting torque. To increase this torque and, at the same time, to reduce the starting current, it has become customary to equip the field structure with a squirrel cage winding buried in the pole structure. This construction also prevents "hunting" and causes the motor to have a greater balancing effect upon the system when the various phases are unequally loaded. This balancing effect of the motor is more fully considered in Experiment 122.

If the motor is started from its alternating current side, a low voltage should first be supplied to the armature. If it is unde-

sirable to draw greater than full load current from the line for starting, this voltage should be made about one third of the line voltage by supplying the motor with power from taps on the transformers or from a starting compensator. To reduce complications, the starting voltage is made one half normal on small machines.

The alternating current supplied to the armature produces a revolving field in the armature structure. This field flux, threading the stationary pole structure, induces a high alternating electromotive force in the field windings. The field windings should either be well insulated or a means should be provided for breaking up the field circuit so that only a few poles are in series when the machine is being brought up to speed. In revolving field machines, this latter method is not possible, on account of complications in construction. It is not necessary in this case, however, as the number of field turns is relatively low. In synchronous converters, on the other hand, the relative number of turns and the electromotive force are both high. Since such machines have stationary field structures, the fields may be switched easily while the machine is running.

If the field circuit is left closed, the starting torque will be very low, since the alternating current flux is superimposed upon a strong uni-directional flux from the fields. To produce appreciable torque, the armature conductors must be so situated relative to the field that a considerable component of the armature current will be in phase with a fairly strong field alternating at the same frequency.

The process of starting is, then, to open the field circuit and to supply a low voltage alternating current to the armature, say one half of the normal voltage. Time should then be given for the machine to attain synchronous speed. Under this condition, the field flux will be supplied by the lagging component of the armature current. The field circuit is then closed and the full field excitation is supplied. If the field is of the wrong polarity before closing the field switch, the machine will slip one pole by

this operation. Full voltage is then supplied to the armature and the field is finally adjusted to give the proper power factor at the terminals of the motor circuit.

When it is inadvisable to start the motor from the alternating current mains directly, a small induction motor may be employed. Power for this motor may be taken from taps on regular transformers, or from a compensator starter. If sufficient direct current power is available, a small direct current motor may be used. In either case, the process of synchronizing is similar to that for polyphase alternators, as explained in Experiment 101.

Data. Start a polyphase synchronous motor by one of the methods explained above. Adjust the field current for unity power factor; also for 80 percent. leading power factor.

No. 103. DETERMINATION OF THE RATIO OF VOLTAGES IN A SYNCHRONOUS CONVERTER.

References. Russell, Vol. 2, pp. 431 to 432; Karapetoff, pp. 536 to 538; Steinmetz' "Elements," pp. 271 to 279; Thompson's "Dynamos," Vol. 2, pp. 502 to 505; Thompson's "Polyphase," pp. 345 to 363; Arnold, Vol. 4, pp. 684 to 686; Franklin and Esty, pp. 180 to 184; Esty, pp. 276 to 282.

Object. To determine the ratio of voltage between the direct current and the alternating current sides of synchronous converters.

Theory and Method. Since the direct current and alternating current sides of a synchronous converter are connected together by the same armature conductors, there is a definite ratio between their voltages for a given wave form and a given position of the brushes. For this experiment, a sine wave will be considered and it will be assumed that the direct current brushes are on the neutral point. Other conditions will be considered in another experiment.

To serve as an aid to the memory, the diagram shown in Figure

103 has been constructed. $ABCD$ is a square the diagonal of which is the direct current voltage. The side of the square is

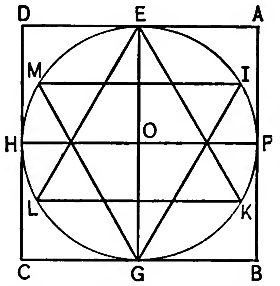

FIG. 103. Diagram for obtaining the ratio between the alternating current and direct current voltage of a synchronous converter.

then the effective value of the alternating current voltage having the diagonal AC as its maximum; or

$$AB = \frac{AC}{\sqrt{2}}.$$

This is also the diameter of an inscribed circle, and hence

$$EG = HF = AB$$

is the corresponding single phase or quarter phase voltage. The voltages between rings in a quarter phase system are, then,

$$EF = \frac{AC}{2}.$$

The side of an equilateral triangle inscribed in the circle is the three phase voltage, or

$$IG = \frac{2OI\sqrt{3}}{2},$$

$$= \frac{EG\sqrt{3}}{2},$$

$$= \frac{AC\sqrt{3}}{2\sqrt{2}}.$$

The pressure between one brush on the commutator and any collector ring is $AC/\sqrt{2}$ when measured by an alternating current voltmeter of the dynamometer type and $2AC/\pi$ when measured by a direct current voltmeter of the D'Arsonval type.

In a similar manner, the double delta, diametral, and voltage between rings may be determined for a six phase machine or a machine of any number of phases, by inscribing the proper figure and applying plane geometry to the solution.

Data. Determine experimentally the ratio, at no load, between the alternating current and direct current voltages of a synchronous converter operated from its alternating current side with the direct current brushes on the neutral. Also determine the ratio of the pulsating voltage to the direct current voltage when measured with the two types of instruments. Shift the brushes slightly each side of the neutral and determine the effect of brush shift. Put the brushes on the neutral and find the effect of loading the machine.

Question. Does the pulsating current wave pass through zero when the load is inductive?

No. 104. TRANSFORMATION FROM DIRECT CURRENT TO ALTERNATING CURRENT BY MEANS OF A SYNCHRONOUS CONVERTER.

References. Arnold, Vol. 4, pp. 706 to 722, and 792 to 798; Thompson's "Dynamos," Vol. 2, pp. 493 to 505; Thompson's "Polyphase," pp. 345 to 363; Steinmetz' "Elements," pp. 277 to 279, 330 and 331; Karapetoff, pp. 539 to 541; Russell, Vol. 2, pp. 416 to 446; Franklin and Esty, pp. 177 to 179; *Am. Elect'n,* December, 1901, A. S. McAllister, Rotary converters.

Object. To operate a synchronous converter from its direct current side.

Theory and Method. A synchronous converter operating under the conditions of this experiment and running without load, is simply a direct current motor. Its field is usually shunt

wound and hence it has the characteristics of a direct current shunt motor.

When the alternating current side is loaded, the armature reactions of this load affect the field flux and hence affect the speed of the machine. On non-inductive load, the armature reactions neutralize each other. On inductive load, the armature reactions do not quite neutralize and the demagnetizing action of the lagging current weakens the field flux and the speed increases. For this reason synchronous converters operating between two systems, should have a speed limiting device, as they are liable to become operative from the direct current side with inductive load on the alternating current side and disastrous results may occur if the machine is not stopped.

Data. Operate a synchronous converter from its direct current side, loading the alternating current side with various types of load.

No. 105. TRANSFORMATION FROM ALTERNATING CURRENT TO DIRECT CURRENT BY MEANS OF A SYNCHRONOUS CONVERTER.

References. Arnold, Vol. 4, pp. 706 to 722 and 792 to 798; Thompson's "Dynamos," Vol. 2, pp. 493 to 505; Thompson's "Polyphase," pp. 345 to 363; Stenmetz' "Elements," pp. 277 to 279, 330 and 331; Karapetoff, pp. 539 to 541; Russell, Vol. 2, pp. 416 to 446; Franklin and Esty, pp. 177 to 179; *Am. Elect'n,* September, 1903, W. F. Fernandez, Operation of rotary converters; *Am. Elect'n,* December, 1901, A. S. McAllister, Rotary converters.

Object. To operate a synchronous converter from its alternating current side.

Theory and Method. When operated from its alternating current side, a synchronous converter has many of the operating features of a synchronous motor. Its field is worked nearer the bend of the saturation curve than that of the ordinary synchronous machine, in order to obtain better commutation. In conse-

quence, a small change in field current will not produce as much change in armature current.

When normally excited, the rating of a polyphase synchronous converter is greater than that of the same machine driven by mechanical power and loaded from either end. A change in field current, either above or below normal, changes the heating, on account of the unbalanced armature reactions. Loading the machine on its direct current side changes the ratio slightly, due to armature drop.

Data. Operate a synchronous converter from its alternating current side with the direct current brushes on the neutral point. Load the direct current side of the machine and determine the voltage regulation with the alternating current voltage and the frequency maintained constant.

Question. What are the relative outputs of a synchronous converter operated single-phase, quarter-phase, three-phase and six-phase?

No. 106. STARTING A SYNCHRONOUS CONVERTER FROM ITS DIRECT CURRENT SIDE.

References. Arnold, Vol. 4, pp. 781 to 784; Thompson's "Dynamos," Vol. 2, pp. 516 to 518; Steinmetz' "Elements," pp. 328 and 329; Karapetoff, pp. 534 to 536; Russell, Vol. 2, p. 439; *Am. Elect'n,* July, 1901, Arthur B. Weeks, Starting rotary converters; *Elec. Jour.,* July, 1905, Arthur Wagner, How to start rotary converters; Standard Handbook, Sec. 6, Art. 182 to 187; McAllister, pp. 168 to 169.

Object. To start a synchronous converter from its direct current side and to connect its alternating current side to a source of power after the motor has been brought up to speed.

Theory and Method. A synchronous converter, when started from its direct current side, is essentially a direct current motor. If the machine is compounded when running under load, the series field should be cut out when starting. The machine is then a

shunt motor and may be treated as such while starting. In many cases where the machine is operated from its alternating current side through transformers, the connections between the transformers and the slip rings are not disturbed during the starting period, the final switching being done on their high tension side. This avoids handling heavy currents on switches and minimizes the deterioration of these contacts.

The field is first connected, all of its control resistance being cut out. Direct current, regulated by some form of rheostat, is then passed through the armature. When the machine has attained normal speed, it is thrown upon the direct current system directly. Speed control is obtained by use of the field rheostat.

The next step is to parallel the machine with the alternating current line. If the machine has previously been operated on the line with the same connections, the phase rotation will be correct. The machine should then be synchronized, as in the case of alternating current generators. Greater care must be taken in this case to have the voltages of the machine and line the same, however, as the alternating current and direct current voltages bear such an intimate relation to each other that inequality of voltage in this case means an immediate transfer of load.

When the machine has been connected to the alternating current line, it may be loaded by raising the alternating current voltage. This is accomplished, in some cases, by the use of a voltage regulator on the alternating current side. In the case of synchronous converters operated in railway service, it is done by a variation of the field current of the machine. This acts upon the line and series reactance in a manner such that the terminal alternating current voltage is increased and, hence, the direct current voltage is raised. Where the regulator is employed, the field rheostat should be used to adjust the power factor at unity or at any other desired value to give the proper operating conditions.

Data. Connect the synchronous converter and start it from its direct current side, paralleling the alternating current side with some source of power. If the machine is newly connected, care

should be taken to see that its phase rotation is the same as that of the alternating current source, before throwing the machines together. After the machine is in operation, shift the load from one side to the other. Vary the field current and note the effect on the load and power factor.

Discussion. Discuss this method of starting, its reliability, speed of operation, and desirability both for railway and for lighting systems.

No. 107. STARTING A SYNCHRONOUS CONVERTER FROM ITS ALTERNATING CURRENT SIDE.

References. Arnold, Vol. 4, pp. 781 to 784; Thompson's "Dynamos," Vol. 2, pp. 516 to 518; Steinmetz' "Elements," pp. 328 and 329; Karapetoff, pp. 534 to 536; Russell, Vol. 2, p. 439; *Am. Elect'n,* July, 1901, Arthur B. Weeks, Starting rotary converters; *Elec. Jour.,* July, 1905, Arthur Wagner, How to start rotary converters; Standard Handbook, Sec. 6, Art. 182 to 187; McAllister, pp. 168 and 169.

Object. To start a synchronous converter from its alternating current side.

Theory and Method. When there is no direct current power available, or when it is undesirable to start from the direct current side, the machine must be started by some other means. One method is to start the machine from the alternating current source of power, using the machine either as a hysteresis or an induction motor. In this case, low voltage taps are furnished on the low tension sides of the transformers and switches are provided for connecting these taps successively to the slip rings of the machine.

In starting from the alternating current side, the field of the machine must be disconnected from the direct current brushes and opened at several points, because of the danger of a breakdown due to high voltage generated in the field windings by the alternating flux from the armature. For this purpose, a "field-break-up"

switch is generally provided and mounted on one side of the machine. This should be opened before starting the machine from the alternating current side.

Half voltage, or less, should first be supplied to the machine and, when it is nearly up to full speed, the other steps should be connected in turn, until full voltage is supplied and the machine is operating at synchronous speed. The field switch should then be closed and the field current adjusted to the proper value.

While the machine is coming up to speed, the field flux has constant reversals, of lower and lower frequency, until finally a constant value is obtained, which is at first supplied by the watt-less current in the armature of the machine and, finally, by the direct current after the field switch has been closed. The fields and, consequently the brushes, may have either polarity, depending upon chance. Before paralleling the direct current side with the bus-bars, it is therefore necessary to know the polarity; for, if the connection is made with the wrong polarity, a bad accident may result. The polarity may be determined by a voltmeter, used either directly across the machine or across the switch to be closed. If the wrong polarity has resulted, the machine should be made to slip one pole by reversing the polarity of the fields. This is usually done by means of the "break-up-switch," which is made double throw for this purpose. After obtaining the correct polarity at the brushes, the machine may be connected to the direct current line and the load may be transferred by manipulation of the field rheostat or by a voltage regulator on the alternating current side.

Data. Connect and start a polyphase synchronous converter from its alternating current side. Measure the current taken to start the machine, both at the brushes and on the high tension side of the transformer, if possible. After the machine is in operation, vary the field current and note the effect of this variation upon the load and upon the power factor.

Discussion. Discuss this method of starting; its reliability,

speed of operation and desirability, both for railway and for lighting systems.

No. 108. STARTING A SYNCHRONOUS CONVERTER BY MEANS OF AN INDUCTION MOTOR.

References. Arnold, Vol. 4, pp. 781 to 784; Thompson's "Dynamos," Vol. 2, pp. 516 to 518; Steinmetz' "Elements," pp. 328 and 329; Karapetoff, pp. 534 to 536; Russell, Vol. 2, p. 439; *Am. Elect'n,* July, 1901, Arthur B. Weeks, Starting rotary converters; *Elec. Jour.,* July, 1905, Arthur Wagner, How to start rotary converters; Standard Handbook, Sec. 6, Art. 182 to 187; McAllister, pp. 168 and 169.

Object. To start a synchronous converter by means of an induction motor.

Theory and Method. Sometimes the conditions of the system are such that the heavy current used in starting a synchronous converter from its alternating current side is very objectionable. This current, besides being large, is a lagging current, and causes a considerable drop in the voltage on the system. When a sufficient amount of power is not available from the direct current side for starting purposes, or where this is also objectionable, the machine is usually started by means of an induction motor directly connected to its shaft.

Power for the induction motor is supplied from the alternating current line, either by means of separate transformers or by means of taps on the transformers used in the operation of the synchronous converter. The converter is brought up to speed by means of the induction motor, this motor being started in the usual manner. There are several methods used in synchronizing the converter, after it is up to speed. One method is to bring it to a speed a little above synchronism and then, cutting off the power from the motor, to synchronize as it slows down to the proper speed.

Another method is to load the direct current side of the machine, a resistance being connected in series with the induction motor. When the slip of the motor is sufficient to bring the converter to the correct speed, it is synchronized. Sometimes the core loss of the converter is sufficient to load the motor. Other methods may suggest themselves. In each of these methods, the motor must have a smaller number of poles than has the converter, in order that its synchronous speed may be greater. The direct current side is then connected as in Experiment 107.

Data. Using an induction motor having a smaller number of poles than the number of poles of the synchronous converter, start the latter by one of the methods suggested above and synchronize the converter with a source of power.

Compare. Compare this method with those in the two preceding experiments as regards reliability, speed of operation and desirability, both for railway and for lighting systems.

Estimate. Estimate the size of motor necessary for starting and synchronizing a given size of machine.

No. 109. COMPOUNDING A SYNCHRONOUS CONVERTER.

References. Arnold, Vol. 4, pp. 706 to 722; Thompson's "Dynamos," Vol. 2, p. 505; Steinmetz' "Elements," pp. 277 to 299; Karapetoff, pp. 541 to 543; Russell, Vol. 2, pp. 435 to 439; Standard Handbook, Sec. 6, Art. 190; *St. Ry. Jour.*, August 29, 1903, P. M. Lincoln, Compound wound vs. shunt converters; *Elec. Wld. and Eng.*, April 9, 1904, F. G. Baum, Synchronous converters; *Elec. Rev. Lond.*, April 13, 1906, Alfred Still, Some notes on the regulation of rotary converters; *Trans. Am. Inst. Elec. Eng.*, May, 1906, W. L. Waters, Shunt and compound wound synchronous converters for railway work.

Object. To study different methods of compounding a synchronous converter.

Theory and Method. Since there is a direct ratio between the alternating and direct current voltages in its armature, a synchronous converter cannot be compounded in the same manner as is a direct current generator. Excepting in the case of the split pole converter considered in Experiment 110, the compounding must be effected by a change in the alternating current voltage supplied. This change may be brought about by the use of a potential regulator in series with the alternating current leads of the synchronous converter. This regulator may be adjusted by hand or by some automatic device operated by the load current. This type of pressure regulation is most commonly used in lighting systems and in other cases where a control of both the voltage and the power factors is required, since the field current of the machine may be regulated to adjust the power factor and the regulator may be adjusted to produce the required voltage.

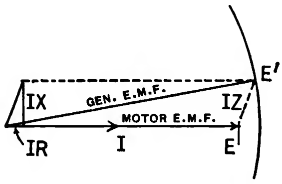

Fig. 109*A*. A synchronous converter at the end of a transmission line, normal excitation.

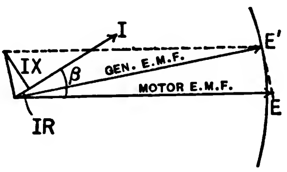

Fig. 109*B*. A synchronous converter at the end of a transmission line, over-excitation.

Another method of voltage regulation is that of relying upon the inductive line reactance and the leading current caused by over-exciting the synchronous converter. In Figure 109*A* is shown the condition for normal excitation when the machine is supplying a given load. Figure 109*B* shows the same load and generator voltage, but a greater value of the terminal voltage *E* of the converter, produced by over-exciting the converter field. The same may be seen in the circle diagram, Figure 109*C*, where *AE'* is the generator voltage and *BE'A* is the no load circle with its center at *O*. *CEA* is the locus of the motor voltage, for constant motor load. It will be seen that over-exciting the motor will cause an increase in the motor voltage *AE*.

With the values of line resistance and reactance so adjusted as to cause the angle θ to assume the proper value, the compounding effect may be made to automatically vary with the load, the point E moving along a locus approximating a circle about A as a center. This voltage may also be made to increase with the load. Compounding may be assisted, in some cases, by supplying the synchronous converter with a series field similar to the series field of a direct current generator. The desirability of this series winding is dependent upon the line constants rather than upon the machine constants, whereas the machine constants govern in the case of direct current machines.

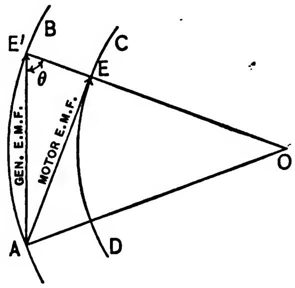

FIG. 109*C*. Circle diagram for a synchronous converter.

Data. Compound a synchronous converter by one of the methods mentioned. For this purpose, it is well to use a single phase converter with an adjustable reactance in series with its alternating current side. Various values of series reactance and field adjustments should be tried, in order to become familiar with the problem.

No. 110. OPERATION OF A SPLIT POLE CONVERTER.

References. *Trans. Am. Inst. Elec. Eng.,* June, 1908, Comfort A. Adams, Voltage ratio in synchronous converters.

Object. To study the operation of a synchronous converter with poles arranged for variation of its voltage ratio.

Theory and Method. The ratio between the direct and alternating current voltages in a synchronous converter depends upon the point in the electromotive force wave where commutation takes place. This ratio may be altered in two different ways.

The flux wave may be shifted with reference to the brushes or

the shape of the alternating current wave may be altered. Each of these effects takes place in the split pole converter. Its poles are split into two or three parts, with independent windings on each. By varying the excitation of each of these parts, the flux wave may be made to shift in position and shape with respect to the brushes which are kept stationary in a good commutating position. The commutation position is possible, due to the slots between the different polar projections. The ratio may be altered to a considerable extent by this method. The heating is changed but little. The efficiency is altered, due to the change in output voltage.

Data. Operate the synchronous converter from its alternating current end at constant voltage and frequency. Vary the field excitation of each of the polar projections and measure the direct current voltage at no load for each change over the entire operating range.

Load the direct current side and keep the current output constant. Vary the ratio over its entire range. Measure the power, current and electromotive force on the alternating current side and the electromotive force and current on the direct current side. Calculate the voltage ratio and observe the sparking of the machine.

Curves. Plot curves illustrating the operation of the machine.

No. 111. EFFICIENCY OF A SYNCHRONOUS CONVERTER BY LOADING.

References. Arnold, Vol. 4, pp. 754 to 756, 823 to 824; Standard Handbook, Sec. 6, Art. 167 to 174, Sec. 19, Art. 137 to 147; *Elec. Eng. Lond.*, December 5, 1902, W. M. Thornton, Experiments on synchronous converters; *Elec. Rev.*, July 9, 1904, E. A. Ryestein, Loading machines for testing purposes.

Object. To determine the efficiency of a synchronous converter, by the output and intake method.

Theory and Method. The commercial efficiency of a syn-

chronous converter, as is true for any electrical apparatus, is the ratio of the power output to the power intake. The efficiency may be determined when power is supplied to either side, the other side being loaded in a suitable manner.

If the power is supplied to the alternating current side, the efficiency should be measured with the field adjusted for unity power factor, unless otherwise stated. The alternating current voltage and frequency should be maintained constant and the intake and output determined for several loads, from no load to 25 percent. or 50 percent. overload.

If the power is supplied to the direct current side, the voltage and speed should be held constant, unless the specifications state that the efficiency and regulation are to be determined under the condition of constant field excitation. Non-inductive load should be used in this test, as inductive and capacity loads cause a greater copper loss in the armature.

Data. Determine the efficiency of a synchronous converter by supplying power to either side and loading the other side.

Curves. Plot a curve between efficiency and percent. load, using efficiency as ordinates.

No. 112. STARTING A POLYPHASE INDUCTION MOTOR.

References. McAllister, pp. 22 to 23; Arnold, Vol. 5, Chap. 12; Esty, pp. 353 and 354; Thompson's "Dynamos," Steinmetz' "Elements," pp. 368 to 372; Karapetoff, pp. 549 to 553; Russell, Vol. 2, pp. 380 to 383; *Elec. Wld.*, March 11, 1909, Selby Hoar, A comparison of the methods of starting squirrel cage induction motors.

Object. To start a polyphase induction motor.

Theory and Method. Polyphase induction motors may be started in any one of several different ways, depending upon the design of the motor, the character of the load and the conditions

of the circuit. A motor having a wound rotor may be started by inserting a resistance in the external circuit of the rotor windings, thereby increasing the starting torque and lowering the current in the stator to a value approximating full load current as a maximum. This resistance is gradually cut out, as the motor comes up to speed. A resistance in the interior of the armature and operated by an external switch, is sometimes used, making the whole self contained. In this type of motor the rotor resistance, when running under normal conditions, may be made low, giving excellent speed regulation.

Another method is to connect an impedance with an iron core, in multiple with a resistance in the external circuit of the rotor. When the rotor is stationary, the frequency of its electromotive force is the same as that in the stator, and the impedance takes but a small current. As the motor comes up to speed, this frequency decreases to a value equal to the slip, which is small under normal operating conditions. The rotor electromotive force is then able to force a large current through the low resistance winding of the impedance. This method is entirely automatic, requiring no switch in the rotor circuit. On the other hand, it is somewhat cumbersome and expensive.

In the case of motors with squirrel cage windings on their rotors, the starting torque is low and the starting current is high. The power-factor is usually higher. It is customary to start this type of motor by the use of a compensator, or lowering transformer, in the stator circuit. This compensator is usually equipped with a switch to throw normal voltage on the machine when it approaches normal speed. This type of motor is generally installed where the starting torque required is below normal full load torque. It is the simplest type of motor to operate.

Data. Connect up motors for operation by each of the methods outlined. In the case of wound rotors, find the value of resistance which will give 25 percent. above full load torque at the start, with normal voltage impressed upon the stator at normal frequency. Find the value and power factor of the stator current,

under this condition. In the case of the motor with squirrel cage rotor, find the lowest stator voltage which will give full load torque at standstill. Find the value and power factor of the stator current, for this condition.

Discuss. The advantages and disadvantages of these methods of starting.

Questions. What type of motor would you use to start a load of 100 horse power, if operated from a 100 kilowatt generator? Why?

What type of motor would you use to drive a constant speed centrifugal pump?

No. 113. STARTING A SINGLE PHASE INDUCTION MOTOR.

References. McAllister, pp. 53 to 62; Arnold, Vol. 5, Chap. 13; Esty, pp. 330 to 333; Thompson's "Dynamos," Vol. 2, pp. 777 to 786; Steinmetz' "Elements," pp. 290 to 297; Steinmetz' "A.C. Phenomena," pp. 380 to 387; Karapetoff, pp. 565 to 567; Behrend, pp. 71 and 72; Russell, Vol. 2, pp. 378 and 379; *Trans. Am. Inst. Elec. Eng.*, September, 1908, I. E. Hanssen, Calculation of the starting torque of single phase induction motors with phase splitting devices.

Object. To start a single phase induction motor.

Theory and Method. Single phase induction motors are not self starting, unless provided with some more or less complicated device for producing a component of the armature current in phase with the field flux. One method is to provide the motor with a commutator and brushes. The brushes are short circuited and set at an angle of about 45 degrees with the field. The motor starts as a repulsion motor. When nearly up to synchronous speed, a device, either automatic or hand operated, throws off the brushes and short circuits the commutator, thus producing an induction motor. This type of motor starts with very good torque and with fairly low current.

Another method is to provide an auxiliary winding on the stator, supplying it with current from some sort of a "split phase" device such as a condenser or a combination of a resistance and a reactance. The machine then starts as a polyphase motor, with most of the characteristics of the same. When up to speed, the auxiliary winding is disconnected (either by hand or automatically) and the machine then runs as a single phase induction motor.

A method used on small motors, is to place a copper band, or shading coil, around a part of each pole. This forms a short circuited secondary and the flux through this portion of the pole has a time lag behind that of the flux in the remainder of the pole. This produces a revolving magnetic field in the iron structure, thus starting the motor. When the motor has attained its normal speed, the current through this winding becomes small and, consequently, the efficiency of the machine is not seriously affected by the introduction of the starting device. Efficiency may be sacrificed for convenience and cost more in small motors than in large ones.

Data. Make starting tests on the single phase motor by one or more of the above methods. Note in each case, if possible, the starting torque, current and power factor. Find also the time necessary for the motor to come to full speed, both for light load and full load torque.

Take a polyphase motor and determine the combination of inductance and resistance, or capacity and resistance, necessary to start the machine under a given load from a single phase circuit. Find the current taken and its power factor during the starting, and discuss the different methods tried. Recommend necessary changes in the circuit or improvements to make the motor satisfactory for operation on a lighting circuit.

No. 114. PRONY BRAKE TEST OF AN INDUCTION MOTOR.

References. McAllister, Chap. 3; Arnold, Vol. 6, pp. 312 and 313, and 328 to 333; Karapetoff, pp. 553 to 561; Russell, Vol. 2, pp. 354 to 358; Swenson and Frankenfield, Vol. 1, p. 252.

Object. To determine the efficiency and regulation of an induction motor by loading with a prony brake.

Theory and Method. The difficulty of separating the losses, and determining their values under different conditions of loading, have led some manufacturers to test their induction motors by means of the output and intake method. In this test the load is applied by means of a prony brake or by a rated generator. In either case the intake is read electrically.

The accuracy of this test is limited by the accuracy of the instruments on the one side and the proper determination of length or losses on the other. The unavoidable errors on both sides makes this test uncertain in its accuracy to a closer degree than about 5 percent. The electrical instruments are much more accurate than the mechanical devices.

By this test the different variables, such as speed, torque, power factor and temperature rise, may be compared with the intake with an accuracy sufficient for ordinary work.

Data. Start the motor and apply different loads by means of a prony brake, from no load to full load, keeping the pressure and the frequency constant. Read the power and the current intake and the brake output for all loads. Make a careful determination of the slip of the motor for all loads.

Curves. Plot the following curves, with percent. output as abscissas:—ampere intake, torque, efficiency, power factor and speed.

No. 115. PRONY BRAKE TEST OF A SINGLE PHASE SERIES MOTOR.

References. McAllister, Chap. 14; *Tech. Qr.*, December, 1905, A study of a single phase series motor; *Zeitschr. f Elektrotech.*, December 27, 1903, M. Osnos, The alternating current series motor; *Elec. Rev.*, February 6, 1904, *Elec. Jour*, March, 1905, February, 1904, F D. Newbury, The alternating current series motor; *Proc. Inst. Elec. Eng.*, April 13, 1905, F. Creedy, The alternating current series motor.

Object. To determine the characteristics of a single phase series motor.

Theory and Method. The direction of rotation of a series motor is independent of the polarity of its source of power supply. On account of this principle the series motor may be used on alternating current circuits. The principal objections to its use on such circuits are poor commutation and low power factor.

Commutation is improved by using low frequency and by reducing the effect of armature reactions by means of a compensating winding. In some motors, resistance leads are used between the armature coils and the commutator segments.

The power factor is improved by using a relatively weak field with respect to the armature. In spite of these precautions, the power factor is still rather low and commutation is poor at certain speeds. The characteristics of this motor are similar to those of the direct current series motor.

No resistance is required for starting an alternating current series motor, since the voltage may be reduced with comparatively little loss by means of a compensator or by a transformer with secondary taps Since no resistance need be used in starting, every point may be made a "running point," which gives a very efficient motor control. It is possible in certain classes of work to save in the size of compensator by providing starting taps from a portion of the winding which is made high resistance. In this case every point is not a "running point."

Data. Connect the motor to an alternating source of power, of the proper frequency, the voltage of which may be varied. Determine the starting torque, current, power and power factor, for various voltages up to the highest safe starting current. If the armature is provided with resistance leads, care must be taken that the armature does not remain in one position long enough to cause a burn out. When the armature is revolving, each resistance carries current but a short time during each revolution.

Applying normal voltage and frequency, take readings of speed, current, power intake, torque and voltage drop across each part of the motor circuit, for loads varying from 50 percent. overload to the load giving the highest safe speed for the motor armature. This may be repeated for other voltages.

Curves. Plot curves showing the complete performance of the motor.

Diagrams. Draw voltage diagrams illustrating the performance of the motor at different loads.

No. 116. PRONY BRAKE TEST OF A SINGLE PHASE REPULSION MOTOR.

References. McAllister, Chap. 13; *Elect'n Lond.,* July 10, 1903, M. M. Latour, The repulsion motor; *Elec. Eng. Lond.,* September 25, 1903, Serial, Will Cramp, Our single phase repulsion motors; *St. Ry. Jour.,* May 28, 1904, G. F. Fauchett, The repulsion motor; *Am. Elect'n,* September, 1904, A. S. McAllister, The repulsion motor; *Trans. Am. Inst. Elec. Eng.,* January, 1904, W. I. Slichter, Speed-torque characteristics of the single phase repulsion motor, C. P. Steinmetz, The alternating current railway motor, February, 1906, M. Milch, Repulsion motor.

Object. To determine the characteristics of a repulsion motor.

Theory and Method. The repulsion motor has many features in its operation similar to those of a series motor. The main difference between the motors lies in the fact that the armature voltage is induced by the field flux in the case of the repulsion motor, while the current is supplied from the outside through the

brushes in the case of the series motor. In the repulsion motor, the armature circuit is completed through the brushes resting on a commutator and which are short circuited. In order to produce a sufficient component of armature current in phase with the field flux, the brushes are set at about 45 degrees with the armature leads lying under the centers of the field poles. This position usually produces the best starting torque and the best operating characteristics.

The commutation is usually poor in a repulsion motor at starting and at low speeds. Between the connection as an alternating current series motor and that as a repulsion motor, the same machine may be operated in various ways for different classes of service and at different speeds. A description of these combinations may be found in recent text books on alternating currents. It is useful to try various combinations and to become familiar with the operation of the machine.

Data. Take readings and draw curves and diagrams for the repulsion motor, as explained in Experiment 115. If possible, try different brush positions.

No. 117. EFFECT OF ARMATURE RESISTANCE ON THE OPERATION OF AN INDUCTION MOTOR.

References. McAllister, pp. 63 to 65; Arnold, Vol. 6, pp. 95 to 97; Steinmetz' "Elements," p. 368; Steinmetz' "A.C. Phenomena," pp. 297 to 302; *Elec. Wld. and Eng.*, November 26, 1904, C. J. Spencer, The induction motor with high resistance secondary.

Object. To test the effect of rotor resistance on the operation of a polyphase induction motor.

Theory and Method. All alternating current motors which have no commutators are essentially constant speed motors. The speed may be controlled only at the expense of some other operating feature. The most common method of obtaining variable speed is by the use of an induction motor with the armature

circuit brought out to collector rings across which variable resistance may be connected. This type of motor is used for fans, pumps, cranes and hoists, where the resistance is used only for short periods.

The introduction of resistance in the armature causes the maximum torque to be produced at a lower speed, but, at the same time, it spoils the regulation and efficiency of the motor. In this respect, the method corresponds to the introduction of resistance in the armature circuit of a direct current shunt motor. In most instances the resistance continues in service only for starting heavy loads and, consequently, the loss is only for a short time. The motor, when operating with its controller in running position, has all this resistance cut out and runs at nearly constant speed.

It is the purpose of the present experiment to find some of the limitations of this method of speed control. The test will be run to determine the regulation, efficiency and torque of the motor, with various resistances in the armature circuit.

Data. Connect the motor to mains of the proper voltage and frequency. Apply different loads to the motor by means of the prony brake and take the following sets of readings:—

1. With the resistance all out of the armature circuit, find the speed regulation and the motor efficiency.
2. Determine the no load speed of the motor for the different notches on the controller.
3. Determine the speed of the motor with full load torque applied, for the different notches on the controller.
4. Find the largest value of torque under which the motor will start, with all of the resistance in the armature circuit.

Compute. The efficiency and the power factor of the motor, for the first condition of loading; the speed regulation and the full load efficiency, for each notch of the controller.

Curves. Plot the following curves for the first condition of loading, with power output as abscissas:—Efficiency, power factor, torque and speed.

Discuss. The limitations of this method of speed control on the basis of the results obtained in the test.

Questions. Is the set of rheostats properly designed for this motor? What effect does resistance in the armature have upon the current in the stator, the output, the speed, the starting torque, the regulation and the efficiency of an induction motor?

No. 118. TEST OF AN INDUCTION MOTOR BY ALEXANDERSON'S METHOD.

Reference. *Elec. Wld. and Eng.*, August 6, 1904, E. Alexanderson, Method of measuring the output of induction motors.

Object. To determine the characteristics of an induction motor by the aid of a direct current generator used as a load.

Theory and Method. The following method of finding the efficiency of an induction motor was devised by E. Alexanderson and has been used by the General Electric Company.

At ordinary loads, the slip of an induction motor is proportional to the torque. The motor is loaded by means of a separately excited direct current generator. The generator is first brought up to normal voltage and the intake of the motor to supply all losses, is determined by wattmeters. The slip is also determined, by some convenient and accurate method. The generator is next loaded, its excitation being kept constant, and its output taken in addition to the previous readings.

Let $P =$ the output of motor with the generator loaded,
$P_1 =$ the losses, due to friction in generator and motor, belt losses, and generator core losses,
$E =$ the generator pressure,
$I =$ the generator armature current,
$R =$ the generator armature resistance,
T and $T_1 =$ the motor torques corresponding to P and P_1,
S and $S_1 =$ the motor slips corresponding to P and P_1.

Then, by assumption,

$$\frac{T}{T_1} = \frac{S}{S_1} = \frac{P}{P_1},$$

or

$$\frac{P-P_1}{P_1}=\frac{S-S_1}{S_1},$$

and

$$\frac{P-P_1}{S-S_1}=\frac{P_1}{S_1}=\frac{P}{S}.$$

Hence

$$P=(P-P_1)\frac{S}{S-S_1},$$

but

$$P-P_1=EI+I^2R,$$

from the direct current readings. Hence

$$P=(EI+I^2R)\frac{S}{S-S_1}.$$

The friction and windage losses of the motor may now be determined in a similar manner. The belt is trown off and the slip S_f determined, running light, corresponding to the torque T_f. Then

$$\frac{P}{P_f}=\frac{T}{T_f}=\frac{S}{S_f}.$$

Proceeding as before,

$$\frac{P-P_f}{P-P_1}=\frac{S-S_f}{S-S_1},$$

or

$$P-P_f=\frac{S-S_f}{S-S_1}(EI+I^2R),$$

hence

$$\text{output}=\frac{S-S_f}{S-S_1}(EI+I^2R).$$

The intake being found by means of electrical instruments, the efficiency may be determined.

For ordinary measurements, this method may be simplified as follows. The losses in generator and belt may be determined and assumed constant for all loads with but slight error.

The generator, normally excited but running under no load, is driven by the motor. Let the losses in the generator and belt be P_2. Then

$$P_2 = P_1 - P_{\prime\prime},$$
$$= \frac{S_1 - S_{\prime\prime}}{S - S_1}(EI + I^2R).$$

Hence, the motor output for any generator load will be

$$(EI + I^2R) + P_2,$$

where P_2 is a constant. The quantity I^2R may usually be neglected.

Data. Drive a direct current generator by means of the induction motor to be tested. Measure the intake of the motor, the slip of the motor and the output of the generator, for each of the following conditions:—

1. The motor running light,
2. The motor driving the generator with normal voltage at its terminals,
3. Various loads on the generator up to the full load of the motor.

Measure the armature and brush resistances of the generator.

Compute the efficiency of the motor by the method outlined, and draw the efficiency curve.

Questions. Compare the accuracy of this method with others employed for the same purpose. Where would you feel justified in using this method?

No. 119. STRAY POWER TEST OF A POLYPHASE INDUCTION MOTOR.

References. McAllister, Chap. 3; Arnold, Vol. 6, Chap. 14; Esty, pp. 355 to 364; Karapetoff, pp. 561 to 564, and 569; *Elect'n Lond.*, March 30, 1906, G. W. O. Howe, The separation of losses in an induction motor.

Object. To separate the various losses in a polyphase induction motor.

Theory and Method. Many of the losses in an induction motor cannot be determined separately. They must, however, be separated from other losses, which are determined for given conditions. It is the purpose of this experiment to ascertain the various losses without load on the motor.

The core loss depends upon the flux density in the iron and upon the frequency. For a given winding, the flux density depends upon the counter electromotive force and the frequency. The copper losses, with the motor running light, may be calculated from the motor resistance and current. The rotor will run

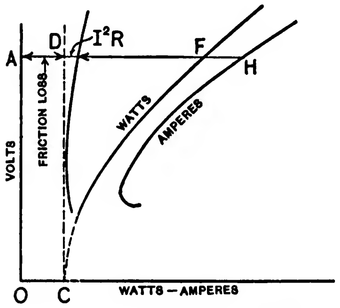

FIG. 119*A*. Curves for determining the friction and core losses, the "running light" current and the power factor.

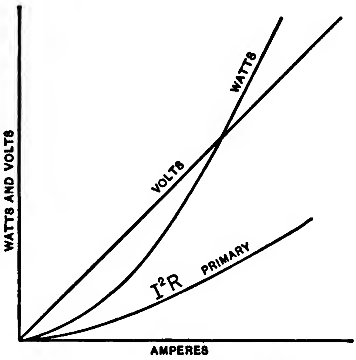

FIG. 119*B*. Curves for determining the copper losses for different armature currents, and the armature current and power factor, at normal voltage and frequency, with the rotor blocked.

with very low voltage on the armature. If it could be run at zero voltage, the only losses would be those due to friction. To separate the friction losses from the "running light" losses, the following method is used. The motor is first run light with different impressed voltages, at the proper frequency. The voltage is varied from a value about 25 percent. above normal to the lowest value at which the rotor will continue to run. The intake in amperes and in watts should be read at the different voltages,

and the results plotted as in Figure 119*A*. The curve between watts and volts, if extended until it cuts the watt axis, will give at this intersection the value of the friction losses of the motor. From *C*, draw a line parallel to *OA*. Let the distance *OA* be the normal voltage, to scale. Draw *AH* parallel to the axis *OC*, intersecting the watt and ampere curves at the points *F* and *H*, respectively. If the I^2R loss for the observed currents is laid off to the left of *CD* as an axis, the part of the abscissa *AF* which lies between this line and the watt curve, is the core loss of the motor at normal voltage.

The rotor is next blocked, and different voltages applied to the stator, varying from zero to a value which will cause a 50 percent. overload current to flow. In this test, the amperes and watts intake of the stator should be obtained and their values plotted as in Figure 119*B*. The curve between amperes and volts will be almost a straight line. The impedance, as determined from this curve, is due largely to the leakage flux in the air gap and teeth. If the curve between the stator I^2R and the current is plotted, it will divide the ordinates of the watt curve into rotor and stator I^2R loss. The watt curve, being due to I^2R losses, will be a parabola and may be extended. Core losses are assumed negligible for the low voltages used in this test. The volts curve may also be extended and the current which would flow at normal voltage with the rotor blocked, may be determined. The power factor under this condition may be found from the relation of watts to volt-amperes.

For these diagrams, current and watts per phase, or total current and total watts, may be used. By total currents is meant twice the current per phase for a quarter phase motor, and $\sqrt{3}$ times the current per line for a three phase motor. That is

$$\text{Watts} = 2EI \cos \beta \quad \text{(quarter phase)},$$

and

$$\text{Watts} = \sqrt{3}EI \cos \beta \quad \text{(three phase)}.$$

If the total current is used, one half of the measured resistance between terminals should be taken for I^2R.

Data. Run the motor light, with different voltages and with normal frequency applied to the stator. Use values of voltages from 25 percent. above normal to the lowest value at which the rotor will continue to run. Take readings of electromotive force, current and watts.

Block the rotor and apply different voltages from zero to a value sufficient to force 50 percent. overload current through the stator. Take readings of electromotive force, current and watts. Determine the stator resistance.

Curves. Plot curves as shown in Figure 119*A*. Determine the friction and core loss of the motor, the "running light" current and the power factor, all at normal voltage.

Plot curves as in Figure 119*B*. Determine the primary and secondary copper losses for different armature currents, and the armature current and the power factor at full normal voltage and frequency, with the rotor blocked.

No. 120. CIRCLE DIAGRAM FOR A POLYPHASE INDUCTION MOTOR.

References. McAllister, Chap. 3; Arnold, Vol. 6, Chap. 6; Thompson's "Dynamos," Vol. 2, pp. 689 to 695; Steinmetz' "A.C. Phenomena," pp. 287 to 289; Karapetoff, pp. 570 to 584; Behrend, pp. 5 to 10; Boy De La Tour, Chap. 7 and 8; Russell, Vol. 2, pp. 343 to 353; Thomälen, Chap. 18; *Zeitschr. f. Elektrotech.*, January 4, 1903, J. K. Sumee, Polar diagrams for induction motors; *Elektrotech. Zeitschr.*, July 23, 1903, A. Heyland, The polar diagrams for the induction motor, November 26, 1903, Ad. Thomälen, A graphical development of the Ossama diagram; *Zeitschr. f. Elektrotech.*, March 13, 1904, H. M. Hobart, Simple comparison methods for the induction motor; *Elec. Wld. and Eng.*, February 25, 1905; H. C. Specht, A practical vector diagram for induction motors, December 23, 1905, W. C. Way, A practical application of the Heyland diagram for induction motors; *Elec. Wld.*, April

21, 1906, A. S. McAllister, Graphic representation of induction motor phenomena, May 26, 1906, A. S. McAllister, Simple circular current locus of the induction motor; *Sib. Jour. Eng.,* November, 1904, A. S. McAllister, The graphic treatment of the phenomena of static transformers and induction motors.

Object. To construct a circle diagram for a polyphase induction motor and from this to determine the operating characteristics of the motor.

Theory and Method. The data necessary for the construction of a circle diagram is obtained from the "running light" and "blocked rotor" tests, as obtained in Experiment 119. These values are determined at full normal voltage and frequency, as explained. No attempt is made here to give the proof for the circle diagram, but a method of construction is developed.

In Figure 120, normal electromotive force ME should be laid off at right angles with the axis MA. Lay off MO to scale, equal

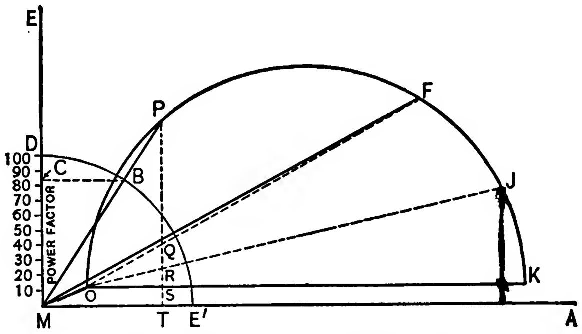

FIG. 120. Circle diagram for a polyphase induction motor.

to the no load current at normal voltage and frequency. It should be laid off at the proper phase angle with ME, by the use of its two components.

Draw OK parallel with MA. Lay off a line, in the direction of MF, from the components of the blocked rotor current determined from Experiment 119. Extend this line, making MF equal to the blocked rotor current at normal voltage. Through the points O and F, pass a semi-circle with its center in OK. This

curve is the locus of the primary current for all loads upon the motor.

Take any value of stator current, as *MP*. The angle *EMP* is the phase angle of the primary current *MP*. *PT* represents the current in phase with the electromotive force *ME*. If multiplied by the electromotive force, it is the power intake. In other words, vertical distances may be taken as watts, to scale. The ratio between the watt and current values is the stator electromotive force. *MT* is then the wattless component of the stator current.

On *ME*, choose some distance *MD* which may be conveniently divided into 100 equal parts. Using this as a radius and with the center at *M*, draw the arc *DBE'*. Through *B*, the intersection of *MP* with this arc, draw *BC* parallel to *MA*. *MC* is then the power factor at that load.

The distance *ST* represents the no load losses of the motor. Let *RS* be taken equal to the additional I^2R losses in the stator for the current *MP*. Draw the straight line *ORJ*. Draw *OF*, cutting *PT* at *Q*. Then *QR* is the secondary or rotor I^2R loss at this load.

Therefore

$$\frac{QR}{PR} = \text{slip in percent. of synchronous speed.}$$

$$\frac{QP}{PR} = \text{speed in percent. of synchronous speed.}$$

$$QP = \text{output of the motor,}$$

$$\frac{QP}{PT} = \text{efficiency,}$$

$$PR = \text{torque developed in synchronous watts,}$$

and

$$\frac{PR \times 7.04}{\text{synchronous speed}} = \text{torque developed in pound feet.}$$

These values may be detemined for different loads and the operating characteristics drawn.

The following may be seen by inspection of the diagram. Maximum power factor occurs for that load which makes the line *MP* tangent to the circle; this is usually about full load. Maximum torque occurs for the load corresponding to a line tangent to the circle and parallel to *OJ*; maximum output occurs for the load corresponding to a line tangent to the circle and parallel to *OF*.

Changing the rotor resistance will not alter either the power factor or the torque until the point *F* reaches their points of maximum value. The speed, output and efficiency are altered by a change in armature resistance.

Data. Determine the values of no load current and watts, short circuit current and watts, and stator resistance, from Experiment 119. Draw the circle diagram. Determine the values of torque, efficiency, power factor and output, for various values of current supplied to the motor.

Curves. Plot the following curves with power output as abscissas for the full range from no load to standstill; amperes intake, torque, efficiency, power factor and speed. Plot a curve between slip and torque, with slip as abscissas, through the whole range of slip from zero to 100 percent. Determine, from the diagram, the maximum values of torque, output and power factor, also the value of starting torque for normal rotor resistance and for a value twice normal.

No. 121. CIRCLE DIAGRAM FOR A SINGLE PHASE INDUCTION MOTOR.

References. McAllister, pp. 114 to 117; Arnold, Vol. 6, Chap. 7 and 8; Thompson's "Dynamos," Vol. 2, pp. 775 to 776; Behrend, pp. 57 to 59; Boy de la Tour, Chap. 7 and 8; Thomälen, pp. 415 to 417; *Elec. Wld.*, June 30, 1906, A. S McAllister, Simple single phase diagrams for single phase induction motors.

Object. To construct a circle diagram for a single phase induction motor.

Theory and Method. The circle diagram for a single phase induction motor is constructed in much the same manner as that for a polyphase motor. In the case of a single phase motor part of the excitation to produce the revolving field has to be delivered to the magnetic circuit by means of the rotor. On account of this fact, the exciting current for one phase of a polyphase motor run single phase is practically double that in the same phase when the motor runs polyphase. It will be seen from this that the "total current" is the same in both cases.

In obtaining data for the diagram, the same process is followed as in the case of the polyphase motor. The motor is first run light at normal voltage and frequency, the amperes and watts

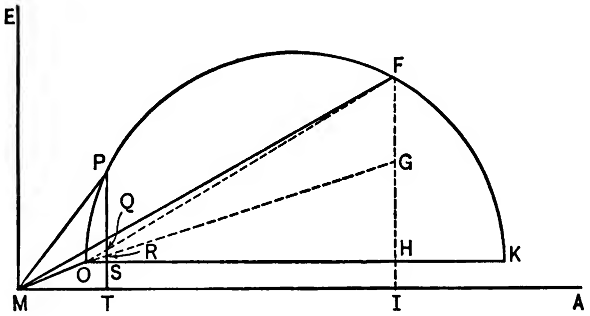

FIG. 121. Circle diagram for a single phase induction motor.

intake being read. The rotor is then stopped and the amperes and watts intake for different voltages are read from a low value up to a voltage sufficient to force 25 to 50 percent. overload current through the stator. From this last set of readings, curves are plotted similar to those in Figure 119*B*. From these readings and curves and the resistance of the stator, the circle diagram may be plotted and the operation characteristics may be determined.

The circle diagram is constructed as shown in Figure 121. The axes *ME* and *MA* are laid off. *MO* is then drawn equal to the no load current at normal voltage, the angle *EMO* being made equal to the no load lag angle. The line *OK* is next drawn par-

allel to MA. MF is now laid off equal to the stator current with rotor stationary and at the proper angle. Through O and F an arc of a circle is passed with its center in OK. This arc is the locus of the stator current for the different loads.

Take P as any point on this locus. Then MP is the stator current and EMP the angle of lag. If a perpendicular PT be drawn to MA, then PT is the intake in watts to another scale, the factor between the watt and current scales being the electromotive force. Draw FO and FI. Take GH on FI equal to the additional copper losses in the stator at standstill. Then FG is the copper loss in the rotor at standstill. The lines FO and GO cut the line PT at the points Q and R, respectively. OK cuts the line PT at S.

Then, at the intake current MP,

$ST =$ core losses and a small stator copper loss,

$RS =$ additional copper loss in the stator,

$QR =$ the rotor copper loss,

$QP =$ the output.

Then

$$\frac{QP}{PT} = \text{the efficiency,}$$

$$\sqrt{\frac{QP}{PR}} = \text{the speed in percent. of synchronism,}$$

and

$$\sqrt{QP \times PR} = \text{the torque in synchronous watts,}$$

or

$$\frac{7.04 \sqrt{QP \times PR}}{\text{synchronous speed}} = \text{the torque in pound feet.}$$

Data. Obtain data as described and construct diagram. Plot curves as in Experiment 119. It is interesting to use the same motor, both single and polyphase, for these experiments and to compare the results. It will be found, if "total current" be used in the two cases, that MF for the single phase machine will be

half that for the polyphase machine, and that MO and MN will be approximately the same in both cases.

No. 122. BALANCING POWER OF A POLYPHASE MOTOR ON AN UNBALANCED CIRCUIT.

Object. To operate a polyphase induction motor on an unbalanced circuit.

Theory and Method. Polyphase circuits are often used for both lighting and power. The lighting load, or any other single phase circuit taking power from the system, is liable to load one phase more than another, causing greater drop in voltage on this phase and the phases become unbalanced.

If a polyphase motor is operating on the system, the motor will take more power from the phase having the highest voltage and it will therefore tend to bring the voltages of the various phases more nearly equal to each other. If the unbalancing becomes great enough, the motor will operate single phase. It will not drop out of step, unless its load becomes too great. The other phases have induced in them an electromotive force equal to the counter electromotive force induced in the motor by the field flux.

If the voltage of one phase drops below this counter electromotive force, due to excessive load or to the blowing of a fuse in that part of the circuit of this phase which supplies power from the generator, this phase of the motor becomes a generator and returns power to any load which may be attached to it. The voltage of this phase will be the motor counter electromotive force minus the impedance drop in the winding. The voltages of the other phases will be their counter electromotive forces plus the impedance drop. There will be a considerable unbalancing in the voltages in such a combination, for non-inductive or inductive loads.

The motor may become heated excessively under this condition without producing sufficient load to open any automatic protective

devices, since the motor must run single phase and carry its mechanical load and, in some cases, an additional electrical load.

Data. Operate a polyphase motor (preferably an induction motor) on an unbalanced system. Determine its balancing power and the additional heating in the motor.

No. 123. CONCATENATION TESTS OF INDUCTION MOTORS.

References. McAllister, pp. 23 to 24; Arnold, Vol. 6, Chap. 20; Steinmetz' "A.C. Phenomena," pp. 319 to 325; Russell, Vol. 2, pp. 368 to 371; *Proc. Inst. Elec. Eng.*, April, 1902, E. Danielson, A novel combination of polyphase motors for traction purposes, June, 1908, H. C. Specht, Induction motors for multiplied service with particular reference to cascade operation; *Elektrotech. Zeitschr.*, January 1, 1903, E. Danielson, A graphical discussion of the cascade operation of polyphase motors; *Elec. Eng. Lond.*, April 1, 1904, E. Danielson, The cascade operation of motors in rolling mills; Steinmetz' "Elements," pp. 423 to 428.

Object. To operate induction motors in concatenation.

Theory and Method. The induction motor is essentially a constant speed machine. Several methods have been devised to give the motor a range of speed. One of these methods is by the use of a resistance in the secondary circuit. This method is open to the objections that there is a considerable energy loss in the resistance and that resistance in the secondary destroys the inherent regulation of the motor. If the load is variable, the resistance must be constantly adjusted to keep the speed constant. This requires an operator and necessitates the employment of a resistance of many steps.

Another method is to bring out a number of leads from the motor winding and, by external connections, to give the motor different numbers of poles. This gives certain definite synchronous speeds for the motor, with good regulation at these speeds. It gives no adjustment between these speeds.

Another method, used in railway operation and in the case of heavy rolling mill motors, is concatenation. Two motors are used, one of which has a wound rotor. One motor has its primary connected to the source of power. The primary of the second motor is connected to the secondary of the first. Both motors are connected to the same load, usually by gears. If the gear ratios of the two motors are the same, they will operate at about half synchronous speed. A speed slightly below synchronism may also be obtained, by connecting the primaries of one or both motors to the source of power and short circuiting the secondaries.

An induction motor is essentially a transformer, with a secondary free to move at different speeds with respect to the primary.

In order to produce torque, a current must flow in the secondary. When the secondary is at standstill, its frequency is the same as that of the primary. When running at synchronous speed, its frequency is zero. Consequently, at half speed, its frequency is half the frequency of the primary. When this is applied to a second motor having its secondary short circuited, the second motor will run at half synchronous speed; that is, at the speed of the rotor of the first motor.

In the first machine, half of the power in the rotor is converted into mechanical power and the other half is given out, as electrical power, to the second motor. In the second motor all the power, except the losses, is converted into mechanical power. Hence, the full power of one motor is derived, at half speed, by the use of two motors.

At full speed the power of both motors is available, or twice the power at half speed. Many types of load require about this ratio of available power. The efficiency of the system is less than that of one motor of twice the size of either motor, but it is greater than that of a motor with resistance in the secondary. When a motor with variable pole connections is used, its exciting current and its regulation are changed with each different connec-

tion. It is also complicated in its windings and control. Neither the resistance nor the concatenation requires the use of a special motor.

Data. Connect two induction motors for concatenation control. Measure the mechanical output and the electrical intake of each motor. Draw the operation curves for the combination, in a manner similar to the method employed for a single motor.

Question. Two like motors are used in concatenation. The gear ratio between the motor and the load is 1 : 1 for the first motor and 2 : 1 for the second motor. What will be the speed of the combination, if each motor has 6 poles and is used on a 60 cycle circuit?

No. 124. EXCITATION CHARACTERISTIC OF AN INDUCTION GENERATOR.

References. McAllister, pp. 81 to 90; Steinmetz' "A.C. Phenomena," p. 312; *Elect'n Lond.,* June 8, 1906, A. S. McAllister, The exciting current of induction motors; *Trans. Am. Inst. Elec. Eng.,* February, 1908, W. L. Waters, The non-synchronous generator in central station and other work.

Object. To determine the relation between the exciting current and the terminal voltage in an induction generator.

Theory and Method. If an induction motor is driven above synchronous speed, it becomes an induction generator. Its excitation, however, must be supplied from an alternating current source. For this purpose, a leading current may be obtained by means of condensers or by the use of an over-excited synchronous machine.

In order to obtain constant frequency, the induction generator must be driven from a prime mover, the speed of which increases with the load. Conversely, if it is driven by a constant speed machine, the frequency varies with the load.

The armature conductors must carry, in addition to the load

current, a current sufficient to magnetize the iron. This magnetizing current is at right angles to the electromotive force. The total current is the vector sum of the magnetizing current and the power current.

Data. Drive the induction generator at normal speed. Connect it across a synchronous generator circuit of the same frequency for the purpose of supplying the magnetizing current. A good plan is to operate the synchronous machine as a motor from the induction generator. Vary the excitation of the synchronous machine, from a value about 25 percent. above normal voltage on the induction generator to the lowest value at which the set will continue to operate. Measure the exciting current and the power in the circuit of the induction generator. Calculate the magnetizing current. A set of condensers may be used for the excitation, if desired.

Curves. Plot a curve between magnetizing current and voltage for the induction generator, using current as abscissas.

No. 125. LOAD CHARACTERISTIC OF AN INDUCTION GENERATOR.

References. McAllister, pp. 90 to 92; Arnold, Vol. 6, Chap. 19; Steinmetz' "A.C. Phenomena," pp. 310 to 319; Steinmetz' "Elements," pp. 407 to 416.

Object. To obtain the load characteristic of an induction generator.

Theory and Method. With constant excitation furnished by a synchronous machine, the external characteristic of the induction generator is similar to that of a separately excited machine. With excitation from a condenser circuit, its characteristic is similar to that of a direct current shunt generator.

Since the excitation comes from the external circuit, all lagging current for an induction load must come from the same source. Hence, with constant excitation, an induction load causes a falling off in voltage. When the quadrature current of the load

becomes equal to the magnetizing current, the voltage becomes zero. A leading current would produce more excitation and cause the voltage to rise.

This experiment may be carried out with constant speed of the prime mover, or with constant frequency from the induction generator.

Data. Excite the induction generator from a synchronous motor, maintaining the terminal voltage normal by varying the field current of the motor. Drive the generator at constant speed. Apply various non-inductive loads to the generator, keeping the field current of the motor constant. Take readings of motor current and power, generator current and power, current and voltage across the load, and frequency. Repeat these observations for inductive and capacity loads of constant power factor. A similar set of observations may be obtained for the condition of constant terminal pressure, the field current of the motor being varied to keep the terminal voltage constant. Other tests may be made by driving the generator so as to produce constant frequency.

Curves. Plot curves between terminal voltage and load current for each condition, using load current as abscissas. Plot curves between frequency (or speed) and load current.

For the test at constant voltage, curves should be plotted between load current and magnetizing current for each load.

Questions. Discuss three methods of shutting down an induction generator. Why is it not necessary to synchronize an induction generator?

No. 126. CIRCLE DIAGRAM FOR AN INDUCTION GENERATOR.

References. McAllister, pp. 75 to 79; *Elektrotech. Zeitschr.*, March 3, 1904, Paul Miller, The circle diagram applied to oversynchronism, August 25, 1904, G. Benischke, The circle diagram for speeds above synchronism; *Elektrotech u Maschinenbau.*,

September 30, 1906, L. Lombardi, Is the circle diagram applicable for induction motors above synchronism?

Object. To derive the load operation characteristic of an induction generator from a circle diagram.

Theory and Method. The circle diagram for an induction generator is but a continuation of the circle diagram for the same machine as an induction motor. Referring to the circle diagram, Figure 126, it will be seen that the semi-circle of Figure 120 is continued into a complete circle. It must be remembered that

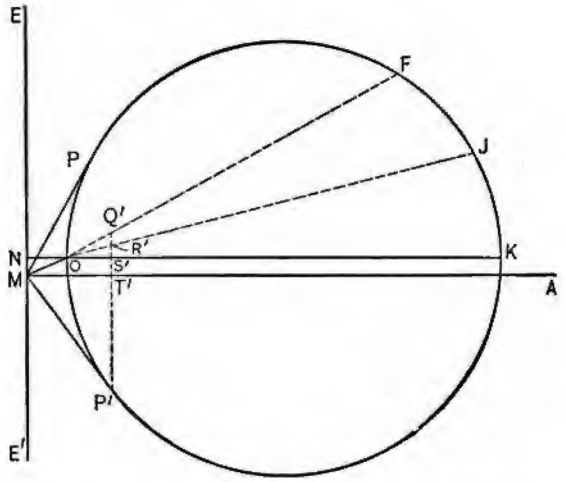

FIG. 126. Circle diagram for an induction generator.

the current at any instant reverses with respect to the induced electromotive force, when the machine changes from a motor to a generator.

Referring to Figure 126, it is seen that the total current is now MP', instead of MP. The output of the machine is now read electrically and is $P'T'$. The intake is the output plus the losses, or $P'Q'$. Hence,

$$\frac{P'T'}{P'Q'} = \text{the efficiency,}$$

$P'R'$ = the intake of the rotor,

$\frac{P'R'}{P'Q'}$ = the speed to produce constant frequency, in percent. of synchronism.

Other quantities may be derived from the diagram, in a similar manner. It is instructive to derive the curves for the full operation of a given machine, both as a motor and as a generator. It is of interest to note that, in the region between the lines MA and OK, the machine is absorbing power, both mechanically and electrically.

Data. Secure data and construct a circle diagram, as described in Experiment 120. Derive data for curves of operation for the machine, both as a generator and as a motor.

Curves. Plot the complete curves, as explained in Experiment 120.

Suggestion. Apply this diagram to the induction generator of Experiment 125 and see whether or not the readings check.

No. 127. TEST OF A MERCURY ARC RECTIFIER.

References. Franklin and Esty, pp. 172 and 173; Steinmetz' "Transient Phenomena," p. 251; *Elektrotech. u. Maschinenbau.*, September 30 and October 7, 1906, A. Liberny, The rectification of currents by the mercury vapor apparatus; *Sci. Am.*, February 17, 1906, A. F. Collins, Mercury arc rectifier for charging storage batteries; *Elec. Wld. and Eng.*, January 17, 1903, Cooper Hewitt, The static converter; *Elektrotech. Zeitschr.*, April 30, 1903, B. Monasch, Pulsating direct current in alternating current arcs; *Trans. Am. Inst. Elec. Eng.*, June, 1905, C. P. Steinmetz, The constant current mercury arc rectifier; *Elec. Jour.*, July, 1905, P. H. Thompson, The mercury vapor converter; *Elec. Rev.*, September 2, 1905, P. H. Wagoner, Mercury arc rectifiers; *Trans. Int. Elec. Congress*, St. Louis, September, 1904, C. P. Steinmetz, The electric arc.

Object. To study the performance of a mercury arc rectifier.

Theory and Method. Any electric arc may be made to rectify alternating currents but the high voltage necessary to produce good results with any other than the mercury arc in vacuum, has made that form of rectifier the most prominent. Up to the present time, only small amounts of power, used generally for charging storage batteries and to operate direct current arc lamps, have been rectified by this means. The synchronous converter is generally used for large amounts of power.

The mercury arc rectifier works on the principle that a current may be made to flow from the positive terminal into mercury, but not in the reverse direction. The mercury must, therefore, be made the negative terminal or cathode of the rectifier. The conductive vapor consists of material thrown off from the cathode. Hence, in order to start the rectifier, this cathode stream must be started. This is usually accomplished by an auxiliary short arc, or by a high potential discharge sufficient to jump across the gap between electrodes.

The current waves overlap sufficiently in the operation of the rectifier, so that the arc may be shifted automatically from one positive terminal, or anode, to another, for each half wave of a cycle. If the arc is broken at the cathode, the rectifier ceases to work until it is started again, because there is then no conducting material in the path of the arc.

The rectifier may be regulated to produce constant current for arc lamp operation, or to produce constant potential for charging batteries. The connections for a single phase rectifier are shown in Figures 127*A* and 127*B*. The alternating current from a transformer, or a reactance coil *AB*, is connected to the two side terminals, or anodes, of the tube. The direct current circuit has its positive terminal at the mercury cup (or cathode) terminal and its negative terminal at the neutral of the transformer or reactance. In order to reduce the pulsations of the direct current, a reactance is often connected in this circuit also.

There is a nearly constant drop in pressure in the rectifier arc, of 13 volts. Hence, when it is used on low voltage circuits, the

efficiency is lower than when operated on higher voltage circuits. The direct current voltage is a little less than half the effective alternating current voltage, since it is less than the mean of the half wave voltage. Or, theoretically,

$$E_{DC} = \frac{\sqrt{2}}{\pi} E_{AC} - 13.$$

Data. Operate the rectifier at its rated frequency and with voltages over the complete range of operation safe for the tube, keeping the current on the direct current side constant at the rated

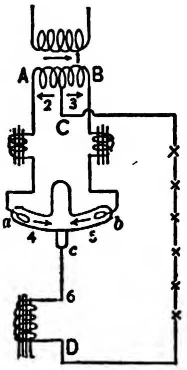

FIG. 127*A*. Mercury arc rectifier producing constant current.

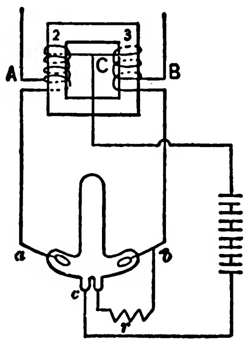

FIG. 127*B*. Mercury arc rectifier producing constant pressure.

value for the tube. Measure the output and intake, voltages and currents and the intake power at the terminals of the tube. Using some constant voltage on the alternating current side, determine the above values for different output currents. Plot curves illustrating the full performance of the tube. If desirable, the reactance or transformer in the primary may be included in the test.

If an oscillograph is available, it is well to study the wave forms of current and electromotive force in the different parts of the circuit. Describe fully the auxiliary apparatus of the rectifier and show the connections by diagram.

APPENDIX A.

THE LIFE HAZARD AND RESUSCITATION IN ELECTRICAL ENGINEERING.*

The vast ramifications of electrical transmission and distributing systems and their remote extensions at high voltages, the overlapping and interlinking of competing properties, and the immense amount of reconstruction made necessary by rapid advances in the art, are tending to make the life of the electrical engineer one of correspondingly increasing danger.

Nor is the public exempt from the penalty of rapid progress, many lives being sacrificed annually on the altar of an art which has not lived up to its responsibilities in the protection of life and property. During the latest period of greatest activity, there were from 300 to 400 deaths per year from electrical causes in the United States.

General Causes of Shock. It is the function of this paper to discuss the sources of danger and causes which lead to injury, the means necessary to prevent or correct them, and the method to pursue in case of injury from electrical shock. The most common sources of serious accident may be enumerated thus:

1. Shocks which the electrical artisan receives due to his carelessness, or that of his co-worker, either on the line or in the station.

2. Shocks which he receives due to imperfections in construction of the line or station which are not avoidable by diligent care on the part of the one injured.

3 Shocks which the public, innocent of danger, receives from outside line construction.

* This appendix contains the major portion of a pamphlet bearing the same title written by Mr. Clem A. Copeland, M.E., of San Francisco, Cal., which is reproduced here with the permission of the author and publisher. The complete copyrighted edition of the pamphlet is published by the Technical Publishing Company, 604 Mission Street, San Francisco, Cal.

4. Shocks which the innocent public receives from inside wiring installations.

General Remedies. By furthering means for the elimination of carelessness and lack of knowledge of danger on the part of the engineer, the corporation and the general public and its employes, and by a proper study and introduction of improved methods of construction and resuscitation, surely nine tenths of all injuries may be avoided.

Some Other Causes. Of all electrical positions, that of lineman is undoubtedly the most dangerous from the standpoint of poor construction, and lack of education in these matters, both of the company and himself.

On account of hasty judgment, we are apt to blame the lineman entirely too much for accidents which occur. Daily contact with extreme danger from an invisible and innocent appearing agency breeds involuntary carelessness. Moreover, if we were all as careful as some, there would be a serious lack of new material.

It is, however, a remarkable fact that the vast majority of linemen know absolutely nothing about resuscitation, and I am not sure but that this applies to other classes of electrical workers. The fault is more blamable, however, in the most dangerous positions.

Telephone linemen are the most careless class of all electricians. This is the natural result of these companies employing men who have never worked with voltages over 50, and amperages above unity, to work amongst circuits of 2,000 to 30,000 volts or more without any knowledge of them, or without having taken the intermediate steps.

The most annoying thing about these matters is, that our own safety depends not only on our own care, or carelessness, but on that of co-workers. A false move of one man destroys the safety of many.

Remedy. A commendable remedy to keep linemen from being careless is for the company physician, or superintendent, or both,

to make trips around the system once per year at least, preferably every six months, and give a demonstration to every "outside" employe as to how to resuscitate a person and call to their attention the various means of precaution against shocks in the first place. The moral effect of this is to make the men more thoughtful, careful and intelligent in these matters. This should apply to telephone companies as well as power distributing concerns.

Every new male employe should have a medical examination and a thorough instruction in resuscitation before assuming his duties. The corporation would thus widely disseminate valuable information for which it would be recompensed an hundred-fold, as well as do a substantial service to the profession.

Primary and Secondary "Crosses." A most common cause of accidents to the public is due to a cross between primaries and secondaries of lighting transformers which may result from defective insulation, throwing the responsibility on the manufacturer of the elements, or a cross on the wires leading to and from the transformer, in which case the distribution engineer may be blamed. As an instance of the latter, a man lost his life by turning on a lamp in a stable while standing on a dirt floor. His heirs were awarded $5,000 damages, which caused a prominent company to remodel many hundred transformer hangings and adopt new and improved methods of distribution.

Those of us who have kept special note of accidents along these gruesome lines will recall several similar cases. In fact this is the most prolific source of public danger, and will be further discussed below.

What Voltage Will Kill. In this discussion, it is pertinent to ask what voltage will kill a person. One may as well ask how many pounds falling on the head will prove fatal. It may be ten ounces or ten pounds, depending on how good a start the weight had before it lit upon the head in question, and to some extent on the head.

I know of an example of the very common case, mentioned above, in which a middle aged, but quite hardy man was severely

shocked by turning on an incandescent lamp while in the bath. The shock was a severe one, and caused him trouble for several weeks. I went twice to measure the pressure the next day. It was 27 volts.

I remember the painful shocks, leaving a taste in my mouth, which I used to receive while repairing a 110-volt direct current two-wire system cable in a wet shaft in the Copper Queen mine, at Bisbee, Arizona, years ago, even when protected by rubber gloves and an umbrella. To have grasped these wires, if bare, while the hands were saturated with the copper sulphate waters of the mines, would have undoubtedly proven fatal.

Perhaps some will remember of the death of a Stanford student, in 1899, who was killed on a 110-volt circuit. On the other hand, I know of three persons shocked by 33,000 volts, one by 15,000 volts, and I think there are cases of direct ground shocks on 60,000 volt lines, which have not proven fatal, while I know of a dozen deaths from 2,000 volts in Los Angeles alone. I have talked with many on this subject, and although we would not advise toying with voltages as high or higher than 30,000, yet it seems that they are as safe or safer than say 2,000 volts.

The higher voltages seem to vent their energy on the surface of the body instead of within—a sort of "skin effect," as applied to a human conductor.

Less Real Danger from High Tension Lines. Along this line there is a curious argument which I have lightly advanced when our municipalities have berated the presence of voltages above 2,000. I have often maintained that high voltages, say 5,000 to 30,000, were less dangerous to life than 2,000; on still another account from that mentioned above. Almost every electrical worker knows the appearance of a high voltage line with its large insulators and bare menacing wires wide apart. He keeps a correspondingly safe distance, and no one ever pretends to work on them while they are "hot," and therefore accidents on them seldom occur. Furthermore, high voltage lines are usually

constructed with more care than the so-called less dangerous or low voltage ones.

Old Question of Grounding Secondaries. With reference to the means to be adopted to prevent injury to the public when turning on incandescent lamps, there is the chance to revive the oft discussed topic of grounding secondaries of lighting transformers, and I venture to advance a few arguments along this line which seem to have been neglected.

In the first place, grounding the secondaries does not entirely remove the danger, for, as we have seen above and we may see from studying many cases, 100 volts may prove fatal, and there is no good reason for disputing that it might thus increase the danger, as it would put a positive ground on the system, enabling one to get 100 volts by turning on a lamp instead of say 27, as in the case mentioned above. One hundred volts in this case would surely have proven fatal.

Moreover, we put the large iron box on the pole and the labyrinth of secondary wires on a dead ground to the 2,000 volt primaries in a most remarkably handy place for the linemen already too hampered with danger. I have maintained practically alone in this matter, that we thus endanger more lives than we protect. In all arguments which I have ever come across, the safety of the lineman has been entirely neglected.

Suggested Remedy. Therefore, I claim that no matter whether the secondaries are grounded or not, the true remedy lies in instructing architects, builders and inspectors not to design nor install nor pass installations, especially lighting installations, in which one may easily touch an electric light socket, switch or other portion of a circuit and at the same time touch any ground, such as a bath tub, wash-basin, telephone, or other signal system or apparatus, cement or dirt floors, gas or water pipes, or grounded metal ceiling or framing, unless a ground-proof or shock-proof switch be installed.

Should this rule be established many lives annually would be saved, such as that of a man in Bakersfield, in October, 1909,

who met death by turning on a lamp while washing a buggy in a stable. This case should not be confounded with a similar one mentioned above, and may prove the fatality of 100 volts under favorable circumstances, as well as the importance of proper regulations as described above.

I should take a neutral stand with reference to the recommendation of the American Institute of Electrical Engineers to grounded secondaries. There are now in use thousands of wiring installations in which a person could easily touch an incandescent lamp and some ground. It is, of course, impossible to remodel all of these installations, but surely the above mentioned rule regulating interior wiring should be observed in future installations, at any rate, for reasons above cited, whether secondaries be grounded or not.

Further Rules. More attention should be directed to the particulars of outside transformer installations and companies should hang transformers with primaries and only the primaries above the transformer on a separate arm, while the secondaries should be specified to go underneath. In remodeling present construction in which primaries and secondaries are run on the same cross-arms as a general rule, the secondaries should dip down under the transformer at the transformer pole. Ten years of careful construction in Los Angeles has proven the wisdom of such a method. As mentioned above, it is the opinion of the writer that primaries and secondaries should never be placed on the same arm, arc circuits being considered as primaries.

It is not the province of this pamphlet to go into details of line construction except in a broad general way, but the above rules should also include the following, as remedies for many other ills:

All transformers setting on the ground or floor, or hanging within ten feet of the ground, must have their cases grounded.

All secondary instruments on high tension work in stations, or elsewhere, must be grounded.

All frames of generators and motors in stations or private installations must be grounded.

All generators and switchboards in stations must be surrounded by suitable insulated platforms.

All guy wires on 2,000 to 5,000 volt work must have strain insulators about six feet from the pole, and must either pull from an eye-rod in the ground, or be grounded by proper methods if pulling from a guy stub, so that the public, as well as linemen, would be protected from shock. I have heard of two serious accidents from neglect of this rule.

On all high tension lines guy anchors only must be used.

All steel poles, if set in cement, must be additionally grounded by approved methods.

All high-tension-line telephones, especially, and in fact all telephones, private or public, should be installed so that the party using them is thoroughly insulated from all grounds, and is unable to touch a ground.

All telephone lines should pass under lighting lines, or be separated by a proper and approved guard. In joint pole work, lighting lines should be placed above and five feet from telephone lines in all cases. I can recall two deaths from disregard of these rules.

All high tension lines should pass above all others.

All pole pins on any cross-arm of either lighting of telephone construction must be at least fifteen inches from the center line of the pole.

No wires carrying more than 110 volts should be allowed to run over a roof where anyone could stand and touch them, either deliberately, or in case of catching at them in slipping.

Companies should prevent any person from engaging in the dangerous side of the profession without proper examination as to general fitness, as to the contents of this paper, and as to physical qualifications. No lineman nor troubleman should climb a pole—either lighting or telephone—who has not passed an examination in these matters.

Other rules which should be enforced by the companies and which are little less important than those above are as follows:

All reels, either pay-out or take-up, should be of iron and set on the ground when in use, and additionally grounded by means of an iron pin driven into the ground and connected to the reel by means of flexible cable. The wire on the reel, if insulated, should be grounded. I know of one life lost and three injuries to as many men which occurred by the neglect of this precaution.

Rules and blue-prints should be prepared, so that lines would be properly guyed, especially at the corners. This is one of the most neglected arts in line work, and many accidents are caused by poorly guyed lines sagging down upon others.

All high tension lines must be shorted and grounded by a chain attached to an iron pin for driving into the ground whenever being worked upon.

All linemen should be required to use rubber gloves and "safeties."

In pulling in or out wires, hand lines should be imperative. I know of one case in which three men were seriously shocked, one of whom died, from the neglect of this precaution.

In house moving or emergency work, no poles should be allowed to stand an abnormal strain without being temporarily guyed. Due to the peculiar way in which poles decay, it is especially impossible to tell whether a pole is safe. I know of one life lost by neglect of this precaution.

Such rules, if we properly push them, lead to a higher dignity of the profession, and a higher appreciation of the dangers in which we work, both from the standpoint of public and corporation recognition.

Assistance by the Profession. Finally, every member of the profession should assist in introducing the customs following:

In all civil service and university examinations there should be questions asked which will show whether the man is familiar with these matters, and if not, he should not be allowed to pass until he has mastered them, no matter how efficient he may be in other lines.

I have been privileged to examine perhaps a hundred linemen,

together with other electrical men, for civil service positions, and have always inserted some such questions as the following: "Suppose one of your fellow workmen should suddenly fall over, shocked apparently to death while working on a 2,000-volt line. Tell accurately, and in detail, just what you would do."

I wish I could publish some of the answers, but it is sufficient to say that 90 percent. of them were radically wrong. Out of one batch of nineteen papers, only one seemed to understand what to do.

I would especially urge that all universities see that their graduating engineers are familiar with these important matters before graduation. They are woefully lax in this regard, and I doubt whether the majority of university engineering graduates can even spell "resuscitation."

In view of the fact that an electric shock from any voltage whatever is simply a case of suspended animation, due to lack of circulation, *i. e.,* a sort of "stunning" of the heart and lungs, as it were, these matters are very important.

Resuscitation. The method to pursue in case one seems mortally shocked by electricity differs only from resuscitation in case of drowning in that the lungs need not be drained of water. Furthermore, the person shocked often swallows the tongue, sometimes rendering resuscitation difficult, which does not occur in the case of drowning.

Resuscitation in case of gas asphyxiation is the same as explained below, although the patient does not swallow the tongue.

The two great and most important factors in electrical resuscitation are:

1. Promptness.
2. Continuity of performance.

I think that within reasonable bounds the method is subordinate, and I am satisfied that more deaths have occurred from lack of knowledge on these two points than from the actual shock itself, or from lack of knowledge of a method. It seems very probable that all deaths could be prevented by prompt and long

continued methods where the shock has not effected a vital wound, or an actual disintegration or change of vital tissue.

I have already mentioned the case in which three men were terribly shocked by 33,000 volts due to pulling in wires without a hand line. One man died as a result of the rest of the party getting "rattled," one man came to life as the result of being badly jolted in an express wagon on the way to the doctor, while the third was saved by a stableman with characteristic "horse sense," who rushed to the aid of the third and resuscitated him by kneading the breast with his knee. These are two cases showing that promptness is subordinate to method.

I know of a similar case in San Francisco, in which a man was grounded on an arc circuit. His face turned black immediately, but he came to while being bounced over a rough road to a drug store; and it seems quite likely that it was the jolting which saved his life. I would further cite the case of a man being shocked in Los Angeles by 15,000 volts, behind a switchboard in a very bad place to work on him. His associate did not yield to the temptation of removing him, but recognizing the value of immediate means, he was worked upon right where he fell without calling for help, as none was near. He was resuscitated in fifteen minutes, showing the value of promptness, while the method in this case was used by one who to my personal knowledge had been well drilled.

On the other hand, I know of eight cases in which men have died in Los Angeles undoubtedly due to a lack of observance of the above points. I believe I am safe in saying that 75 percent. of all deaths from shocks could have been prevented by promptness of performing some method, and half of these are due to lack of any knowledge of when and what to do by co-workers of those shocked. The other half are due to disturbing the patient while being worked upon to put him in the emergency wagon when he is carried to the hospital, and of course dies on the way. There is little chance of properly or continually working on a man in these close wagons, and I do not think there are

FIG. A1. Arms compressing the chest at lower ribs to expel the air—limbs extended (see fifth note on Method).

many who can cite an instance of a man being saved by such a means. He may have revived in spite of it.

In the *Journal of Electricity, Power and Gas* for October, 1902, page 195, you may find the following note by Mr. Lighthipe:

"Sometime ago there was a man tied up on a pole on the corner of Eddy and Mason streets (San Francisco) with a great crowd around him, and four or five policemen guarding the bottom of the pole waiting for the ambulance to come. They finally did get the man on the sidewalk and there he lay. They would not let anyone approach him. Although I tried to get to him and informed them as to my identity, I was promptly ejected. He lay there by actual count, twenty minutes, before the patrol wagon came and carried him away to the city hall, where he was pronounced dead. . . . The policemen immediately assumed that the man was dead because he was motionless."

The remedy is patent. Those having the direction of emergency wagons should be instructed as to how to work on those shocked, and they should be instructed not to remove one being properly worked upon when the wagon arrives, or they should know how and take charge of the resuscitation on the spot without removal in case they are not being worked upon properly, or at all.

Notes on Method. The following illustrated notes will explain the method to be employed:

1. Chop wires with hatchet on block of wood or pull man off circuit by coat-tail or loose clothing, not by his shoes, for they have dangerous nails. Clothes next to the body are not insulators on account of perspiration, so look out and don't get shocked yourself.

2. Tear off any obstructions around throat, as collars or handkerchief, shoulder-braces, or suspenders, and loosen clothing. Don't expose the breast any more than necessary. Heat of the body should be conserved as much as possible to help circulation.

3. Put him in such a position that the head is lower than the feet if possible and handy.

4. If you are the only one present to resuscitate the man, don't send for anyone, or anything, but do it yourself quickly, and don't get "rattled." Lay man on his back and commence resuscitation method below immediately, right where he falls, and don't miss a stroke until he is revived, if it is possible to do so, and don't let anyone remove him to new location. Every fraction of a minute is valuable; one does not know what second life may become extinct.

5. Compress the chest at the lower ribs, then extend arms back over the head in the plane of the body and then bring them down forward over the chest and compress the chest again, repeating the process deliberately about sixteen times per minute to duplicate natural breathing.

6. You should hear air freely issuing from throat when ribs are compressed. If not throat is probably plugged with tongue. Stop long enough to force open mouth with pliers or stick of wood (not your fingers, you may be bitten and delayed). Then pull tongue out a ways with handkerchief or pliers, stick a pin, safety-pin or piece of wrapping wire through it to hold it out, then commence resuscitation method.

N. B. (*a*) Don't put fingers in mouth unless it is blocked open.

N. B. (*b*) Rigidity or resistance to the resuscitation method is a good sign.

N. B. (*c*) Vomiting is also a good sign, but you will have to clean out throat a little with fingers or it may obstruct breathing. You might have to turn man over on face for an instant and lift up waist of body to drain the throat. Return to resuscitation method as soon as possible.

7. If there is one other man present get him to roll up a coat and put it between the shoulders. If a coat is not handy take a block of wood, or a flat stone, but don't let him interrupt the process of resuscitation.

8. Also get him to force open the mouth and draw the tongue out a little ways, as explained in sixth note above, keeping throat clear. While he is doing this use the method shown in

FIG A2. Arms being extended back over the head in the plane of the body (see fifth note on Method) and knees being pressed up towards the breast (see eleventh note on Method).

FIG. A3. Arms fully extended over head, knees up to the breast.

[redacted]

Fig. A5. With the fingers spread out wide grasping the lower chest and with the base of the hand on the diaphragm, deliberately and powerfully press inward and upward toward the heart and then slowly release the pressure. This is repeated deliberately sixteen times per minute to duplicate breathing. Elevate the feet if possible.

9. Then also get him to run to nearest house if not more than a half block away and get blankets to cover him up so as to conserve heat of body. Warm weather may be uncomfortable for those working but is favorable to the man shocked.

If shocked in freezing or snowing weather it may be advisable to take him into a warm house if very near by, in violation to fourth note or instruction above, if it can be done quickly.

10. If there are two present to help, send one for a doctor with an oxygen tank if possible. Do not send for the emergency wagon until he shows signs of life.

11. Have one of your helpers lift up legs, bend them at the knees and press the knees up towards the breast, then bring them back down and straighten them out. Perform this in unison with the resuscitation method, so that the knees follow the arms back and forth, as shown in Figs. A1, A2, A3 and A4.

12. Don't give up working, no matter how long, unless eyeball is soft. Compare your own eyeball with the injured man's. He may revive in five minutes or several hours; usually from one to four hours.

13. Don't try to administer whisky or water or anything else until the man is revived and stands up or sits up.

Treatment of Injury from Burns. 1. If the clothing is on fire smother the flames with your coat or a blanket or roll person over and over in the dirt.

2. Send for a physician.

3. If burn is very severe and physician cannot come immediately, remove only the clothing which does not touch the burn and then lower patient into bath tub of lukewarm water in which a package of baking soda has been dissolved, by means of a sheet

used as a large sling, or some such means, allowing him to remain thus for about half an hour.

4. If physician is not then available cut away the clothing around the burn. Don't try to peel the patch of clothing off the burn if it sticks. Dry off the burned place by delicate touches of absorbent cotton gauze or soft linen. Then saturate the burn thoroughly, or immerse it, if possible, in an emulsion of half and half limewater and boiled linseed oil, made by violently shaking them together in a bottle ("carron-oil").

If limewater is not at hand use boiled linseed oil alone, and if it is not handy use raw linseed oil, olive oil, sweet oil, or as a last resort, vaseline.

Bind up burn with absorbent cotton saturated with carron-oil, and loose gauze or linen bandage.

5. If the burn is moderately severe remove clothing which does not touch the burn, then cut away the garments around the burn which touch it, being careful not to peel anything off which sticks to the spot, and if possible immerse burned spot, and patch, in limewater for half an hour or so. If not, bathe it in limewater for an hour. If no limewater is at hand use baking-soda-water in the same way. Follow by drying off and then saturating burn with carron-oil. Wrap up burn with absorbent cotton saturated with carron-oil, with loose gauze or linen bandage.

Limewater and boiled linseed oil are very important remedies in case of burn, either slight or severe. Every station and line crew should have at least a quart of each on hand. In the station a gallon of limewater and a quart of boiled linseed oil and an empty clean quart bottle should be kept on hand for immediate use.

Flash in the Eyes. Should one become injured by a flash in the eyes due to the blowing of a fuse, the "slop-over" of a direct current generator, the opening of a switch under excessive load or a short circuit, apply a cold compress to the eyes consisting of absorbent cotton, thin gauze, or thin linen saturated with boracic acid formed by dissolving a teaspoonful of the crystals or powder

Fig. A4. Arms being brought down forward over the chest.

Fig. A5. Alternative Method.

in a cup of hot water. Remain in a darkened room for some time, keeping the pack on continually. Use blue glasses when able to go out in the light, bathe the eyes several times a day in boracic acid, and wear the pack at night until recovery.

Poisonous Bites. Many electrical industries are carried on where there is danger from poisonous reptiles, such as rattlesnakes and Gila monsters, and from insects such as tarantulas and centipedes. If bitten, cut several small gashes radially from the bite and suck out the blood for a short interval of time. Then force in as far as possible and as much as possible of the crystals of permanganate of potash and let them dissolve. One must be careful not to cut an artery in so doing. Bind up the limb, or finger, between the bite and the heart very tightly with a twisted handkerchief, or piece of cloth, or soft rope, to keep the blood from circulating freely in the portion containing the bite. The author has known of two lives which were saved by this remedy, now recognized as a standard.

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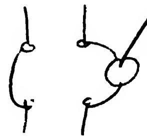

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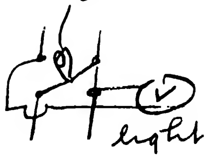

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